Fiscal federalism, local public works and corruption[*]

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Abstract

In ¯scal federal systems, local public works that generate spillovers are often ¯nanced by transfers from higher levels of government. In this article, we consider a governmental hierarchy composed by a Federal and a Local Government. The former delegates to the latter the task of ¯nding a ¯rm to undertake a local public work. As the Local Government has more information about the e±ciency of the ¯rm, it communicates its private information to the Federal Government which decides the way to fund the project.

If side-contracts between the local authority and the ¯rm in charge with constructing the project are feasible, di®erent stakes for collusion may arise. The Local Government can overstate the e±ciency of the ¯rm to obtain extra rents or can also understate it, to ensure the e®ective undertaking of the project. In order to ¯nd the optimal allocations, we derive a \Collusion-Proofness" property which states that, in order to maximize its expected welfare, the Federal Government can restrict to o®er collusion-proof contracts. These contracts indicate the transfers between the Federal Government, the local authority and the ¯rm of constructors. Finally we characterize the distortions set to attenuate the resulting implementation costs. They concern the cost of the project and the decision about its e®ective undertaking. The most important result of this article is that the undertaking of useless projects, at an in°ated cost, is an optimal response of the Federal Government to the threat of collusion between the local authorities and the ¯rm of constructors.

1 Introduction

In ¯scal federal systems, local public works that generate spillovers are often ¯nanced by grants-in-aid from higher levels of government. When a Federal Government delegates to a local authority both the decision about the undertaking of such projects and the search of a ¯rm of constructors, the Local Government can try to obtain more funds than necessary. Those extra funds are either private bene¯ts for the bureaucracy or jurisdictional gains for the constituency. For example, if local authorities know better than the Federal Government the information concerning the bene¯ts of a particular local public work, they can manipulate their private information during the administrative procedures that allocates federal grants-in-aid. But, if the information concerns the cost-side of the project, there is also room for collusion or corruption between Local Governments and the ¯rms of constructors to obtain extra rents.

The second issue is an important empirical problem. Some evidence suggests that in°ated infrastructure costs or useless local public works are the consequence of collusion between local authorities and managers of ¯rms of constructors. Although it is well known that this phenomenon is widespread in developing countries, some industrialized nations have also confronted problems of this kind. The French magazine Le Point (1997) quoted, in an article describing some striking local projects, \white elephants also exist in France".[1]

To deal with those issues, we study a governmental hierarchy, where a Local Government ¯rst chooses a public work and a ¯rm in charge with constructing it. Then, in order to obtain funds, the local authority presents the project to the Federal Government because the latter is not able to observe ex ante its cost. The model presented here is a straightforward extension of our previous article (see Besfamille (1999)). There we analyze the impact of local interests and multidimensional asymmetric information on the design of grants-in-aid that a Federal Government sets to ¯nance local public works with spillovers. We characterize the optimal contract o®ered by the Federal Government to the local authority and to the ¯rm of constructors. The retained formalization enables us to obtain some interesting results, specially the distortions concerning the federal decisions wether to undertake the local project, distortions with respect to the full information framework. These distortions imply that, depending on the possible values of the local bene¯t of the project, more or less local public works are constructed.

But in our previous article, side-contracts between the Local Government and the ¯rm of constructors were infeasible. Here we relax this assumption hence both the Local Government and the ¯rm can collude against the objectives of the Federal Government. Although we maintain an assumption adopted in vari-

[1] The expression \white elephants" refers to large but useless projects (such as snow-plows sent to Guinea) that were undertaken on behalf of the international ¯nancial institutions during the sixties and the seventies in Africa.

ous articles that apply the hierarchical-contractual approach to collusion, namely asymmetric information on the e±ciency of the ¯rm, the present article extends some results of the existing literature.[2] Most of those mentioned articles generally assume that projects are always done and the supervisors in the hierarchies have no real interest on them. Therefore those articles concentrate the attention only on the problem of cost-padding. As we also relax both assumptions, by endogenizing the decisions about the undertaking of a project in the jurisdiction of an interested authority, we are able to describe and analyze the following situation, which represents a di®erent misbehavior from those already considered. In order to obtain grants to undertake a low (but non zero) valued project in its jurisdiction, a local authority can negotiate with the ¯rm of constructors and then present to the Federal Government a project for a local public work with a high rate of return, by underestimating its real costs. Therefore an issue related to the well-known problem of costs-overruns appears, but in a static framework without any consideration of the Federal Government's lack of commitment.

We derive a \Collusion-Proofness" property to characterize the optimal allocations. This property states that, without any of generality, the Federal Government can restrict itself to o®er collusion-proof contracts. Those contracts include the decision about the undertaking of the project, a target cost if the work should be done and the transfers to the Local Government and to the ¯rm of constructors. As usual in the models with collusion and cost-padding, when the project is undertaken, the target cost is generally distorted with respect to the ¯rst-best. But the distortion is not the same if the project is always undertaken or if it is constructed only by an e±cient ¯rm. In the ¯rst case, the well known upward distortion for the ine±cient ¯rm emerges. But in the other case, the Federal Government sets a target cost for the e±cient ¯rm that is lower than the ¯rst-best. In order to attenuate these distortions, the Federal Government makes di®erent kind of transfers to the Local Government: either conditional non-lineal grants-in-aid if the project is done or positive compensations in case of shutdown. Finally the Federal Government decides to distort the decisions that concern the undertaking of a particular project. The most interesting result states that useless projects are nevertheless undertaken at an in°ated cost.

This paper is related to di®erent recent literatures. First of all, we adopt a contractual approach to analyze the allocations in decentralized organizations, as advocated in Caillaud, Jullien and Picard (1996) and Cremer, Estache and Seabright (1996). We also ¯x the organizational framework, similar to the \regulatory capture" approach. As in La®ont and Tirole (1991 and 1993), we analyze a hierarchy. But unlike them, we do not allow the top level of the hierarchy to communicate with the lowest level (in our case, the ¯rm). Because there are

[2] This literature started with the book of Rose-Ackerman (1978). The ¯rst attempt to formalize rigorously her intuitions appeared in the seminal article of Tirole (1986). For a recent survey of the contractual approach to collusion in organizations, see La®ont and Rochet (1997).

costs in the channels of communication, the Federal Government delegates to the Local Government the task of ¯nding the best projects for its jurisdiction and the most e±cient ¯rm to undertake them. The local authority is hence an \intermediate" type of supervisor. We qualify it in this way because, as the Local Government is interested in the e®ective undertaking of the project, it is neither the neutral supervisor depicted in the auditing literature nor the productive one as formalized in the articles that analyze the problems of delegation.[3] In fact we can say that the local government behaves like the agent in the model of Aghion and Tirole (1997). As was indirectly quoted above, we will not try to ¯nd the optimal organization in the presence of the threat of collusion.[4]

We analyze corruption using an informational and contractual approach. Following Tirole (1986), the Local Government will be asked to report its private information in order to enable the Federal Government to implement the ¯nal allocation. In this setting, corruption is formalized as the result of side-contracting between the Local Government and the manager of the ¯rm of constructors. This side-contract would stipulate how the Local Government should manipulate and misreport and then the covert-transfers to set between the corrupt agents. Although our model is an \incomplete-contract" one (because of the broken communication between the Federal Government and the ¯rm of constructors), we are able to prove a \Collusion-Proofness" property. Therefore, unlike Kofman and Lawarr¶ee (1996), an incentive-compatible collusion-proof allocation dominates in equilibrium.

Finally our paper is also related with a growing literature on incentives and ¯scal federal systems. Although the ¯scal federalism approach is based on informational issues (see Oates (1972)), only recently some authors have rigorously study the incentive problems that emerge in such organizational frameworks, among them Gilbert and Picard (1996), Cremer, Marchand and Pestieau (1996), Bucovetsky, Marchand and Pestieau (1998), Lockwood (1999) and Boadway, Horiba and Jha (1999). Even if none of these consider the problem of corruption at the local level, the informational structure of our model and some results are similar to them. We endogenize the structure of the transfers within the hierarchy, like Cremer, Marchand and Pestieau (1996) did. Although our results do not concern the same type of agents and are more general, we ¯nd the same costs distortions than Boadway, Horiba and Jha (1999). Moreover we obtain exactly the opposite distortions than Lockwood (1999). Finally we observe under and over-construction of local public works, like Bucovetsky, Marchand and Pestieau

[3] On one hand, the auditing literature followed Antle (1982) by assuming that the person in charge of controlling the productive agent takes only in account his retribution, independently of the action decided by the principal. On the other hand, the delegation approach as set in Baron and Besanko (1992) and Melumad, Mookherjee and Reichelstein (1992 and 1995) formalizes the supervisor as a productive agent.

[4] There are some recent articles that analyze this issue, as La®ont and Martimort (1996 and 1998) and Bardhan and Mookherjee (1998).

(1998) and Lockwood (1999), again in the opposite way with suspect to Lockwood's results.

The structure of the paper is as follows. In the next section we describe the model and its timing. Section 3 presents the benchmark: the optimal contracts when collusion is infeasible. Next we discuss about the possibility of collusion and we prove a "Collusion-Proofness" property. In Section 5 we find the cost-minimizing collusion-proof contracts. In Section 6 we show the optimal contracts and then we conclude. All proofs are shown in the Appendix.

2 The model

We consider a national government as a hierarchy, whose highest level is the Federal Government (FG) and then comes below a Local Government (LG). The Federal Government must decide, following a report made by the Local Government, whether to finance a local public project. If the project is undertaken, the total benefits for the population are $NB = LB + SB$ where $NB; LB$ and SB stands respectively for "national", "local" and "spillover" benefits. The last two values are strictly positive and perfectly separable between the region where the project is to be done and the rest of the country. Both are common knowledge.

The unique candidate to undertake the project is a firm of constructors (F). If the project is done, its ex-post observable cost is $C = \theta - e$, where θ is an efficiency parameter (the firm's type) and e is the effort exerted by the firm to reduce costs. For the sake of simplicity, we assume that $\theta \in \Theta = \{\theta_l; \theta_h\}$ and $\Delta\theta \equiv \theta_h - \theta_l > 0$. The firm faces a monetary equivalent cost of effort $\psi(e)$. The function $\psi : \mathbb{R} \to \mathbb{R}$ is only defined over the positives and is always positive, strictly increasing and convex. In order to ensure an interior solution, we assume that $\psi(e) = 0$ and $\lim\limits_{e \to \theta} \psi(e) = +\infty$. Moreover $\frac{\psi''}{\psi'}$ is strictly increasing, which implies that $\psi''' > 0$. We adopt an accounting convention that the cost C is totally reimbursed by FG. The utility of the firm is

$$U \equiv t + b - \delta\psi(e)$$

where $\delta = 1$ if the project is undertaken and 0 otherwise. t is the net transfer received from FG and b is a side-payment from LG. The firm's reservation utility level is normalized to 0 so its participation constraint is $U \geq 0$.

The Local Government is benevolent with respect to its constituency. LG knows the firm's type θ and its task is to make a report to FG. As people living in this jurisdiction enjoy LB, the local authority gains from the undertaking of the project. There are also monetary transfers s between FG and LG. We do not impose a priori neither the form nor the direction of these transfers. When $s \geq 0$, FG compensates the region. As the local authority has the power to levy taxes, FG can impose $s < 0$ to LG. Moreover, the Local Government can also make transfers to the firm of constructors, transfers that can be either positive or negative. So LG's utility function is

$$V \equiv \delta LB + v(s - b)$$

where the strictly increasing, concave and differentiable real-valued function $v(x)$ captures the impact of both kind of transfers on the local welfare. We assume that $v(0) = 0$ and $v'(0) = 1 + \lambda$; where $\lambda > 0$ is the shadow cost of public funds raised by all other jurisdictions. The retained formalization for v is an approximation that captures the most important features of the public finances of the region where the project is to be undertaken. LG's participation constraint is $V \geq 0$. This constraint reflects implicitly that FG can prohibit LG to undertake the project by himself.

The Federal Government seeks to maximize the national social welfare and wants the project to be undertaken provided that it has a positive social value. But FG is unable to distinguish between the two components of the cost C: In order to fill this informational gap, FG must rely on a report made by LG. The fact that both levels of authority represent different populations will create a conflict of interests. To deal with, FG faces a mechanism-design problem. FG offers to LG a public works contract: a pair

$$\{M; y(\hat{m})\}$$

which specifies the space of messages M to send (the reports) and the final allocation

$$y = \begin{cases} (\delta = 1; C; t; s; \pi) \\ (\delta = 0; t^{o}; s^{o}) \end{cases}$$

as a vectorial function of the LG's report $\hat{m} \in M$. π is a penalty for LG when the project is accepted by FG but rejected by the firm (see the timing below). The social welfare criterion is given by

$$\begin{aligned} W &\equiv \delta[SB - (1 + \lambda)C] - (1 + \lambda)(t + s) + U + V \\ &= \delta[NB - (1 + \lambda)(C + \Psi(e))] - \lambda U - d(s - b) \end{aligned}$$

where $d(s - b) \equiv (1 + \lambda)(s - b) - v(s - b)$ is the deadweight loss function generated by the inter-jurisdictional transfers s and the side-payments b. FG dislikes to leave any extra rent to the manager of the firm because $\lambda > 0$.

The timing of the model is as follows

1. Nature randomly selects θ: F and LG observe this value.

2. FG designs and offers the public works contract to LG.

3. Collusion between LG and F may take place.

4. LG either rejects or accepts the public works contract. If, on one hand, LG rejects, the game ends. LG and F get their reservation utilities. On the other hand, after accepting, LG must report to FG. Then the latter decides, by imposing $\delta(\hat{\theta})$; if the project should be shutdown or undertaken.

   (a) If $\delta(\hat{\theta}) = 0$; $t^0 = 0$ and $\hat{s}^0 = s^0(\hat{\theta})$ are made.

   (b) If $\delta(\hat{\theta}) = 1$; FG communicates to LG the corresponding cost-transfer scheme to offer to the firm (i.e. the couple of values $\hat{C} = C(\hat{\theta})$ and $\hat{t} = t(\hat{\theta})$). F can refuse or accept this proposal.

      i. If F refuses, FG imposes the penalty π to LG: a fine $f > 0$ and the shutdown of the planned project.[5]

      ii. If F accepts, the project is undertaken and all transfers are made.

As we can see, there is no direct communication between the Federal Government and the firm because the latter does not report its type. F only announces publicly if it accepts or refuses the cost-transfer scheme $(\hat{C}; \hat{t})$; perhaps after a covert negotiation with LG. This seems to be a realistic assumption because, for local public works, there is usually no communication between the central government and the firm in charge of the construction. This reflects "decentralization" in a contractual sense, as set in Caillaud, Jullien and Picard (1996): FG delegates to LG the search of the firm to undertake the local project.

But more important is the consequence of the possibility of side-contracting and the retained timing. In our previous article where side-contracts are infeasible (see Besfamille (1999)), there is already a trade-off in the incentives of LG to misreport the type of the firm. Because LG is indeed interested in the effective undertaking of the project, it might be tempted to make a report to induce it. Nevertheless, it is limited by the fact that, when a project is accepted by FG, the cost-transfer scheme $(\hat{C}; \hat{t})$ to offer to firm depends on the report $\hat{\theta}$: And, in case of refusal by F, LG is penalized. But in this model, LG can attenuate this trade-off by coordinating his announcement with F.

We adopt some methodological assumptions. The first one is common in incentive theory: the full commitment for the public works contract. The other two assumptions are more specific. We assume that FG can imperfectly control the communication between LG and F. On one hand, when FG accepts to undertake a project and communicates to LG the cost-transfer scheme $(\hat{C}; \hat{t})$; LG can not propose to F a different cost-transfer pair. On the other hand, side-transfers (or their monetary equivalent) between LG and F are feasible and can not be controlled by FG. Therefore, in this model, the only way for LG to misbehave

[5] In this case, F gets its reservation utility.

is trough its reporting strategy, reporting strategy that may be the result of a covert negotiation with F.

This paper analyzes the optimal contract that FG offers to LG to obtain its private information. We present two useful benchmarks: the full-information contracts and the contracts that would have arisen under asymmetric information on the firm's type θ but when collusion is not feasible. Then we characterize the optimal contract under asymmetric information on θ and collusion.

3 Optimal contracts when corruption is infeasible

In order to obtain the benchmarks, let's assume that side-contracts are not feasible. Then if FG knows θ and can also observe e, the target values are: e^*, $C_l^* = \theta_l - e^*$, $C_h^* = \theta_h - e^*$, $t^* = \psi(e^*)$ and $s^o = s^* = 0$. The optimal allocations under full information are characterized as follows.[6]

Proposition 1 *Suppose that the Federal Government knows the firm's type θ and can also observe the effort e. Then*

- *when $\theta = \theta_l$, the Federal Government optimally sets*
  - $\delta = 0$, t^o and s^o if $LB < LB^*_{Inf}$
  - $\delta = 1$, C_l^*, t^* and s^* if $LB \geq LB^*_{Inf}$

    where $LB^*_{Inf} \equiv (1+\lambda)(C_l^* + t^*) - SB$
- *when $\theta = \theta_h$, the Federal Government optimally sets*
  - $\delta = 0$, t^o and s^o if $LB < LB^*_{Sup}$
  - $\delta = 1$, C_h^*, t^* and s^* if $LB \geq LB^*_{Sup}$

    where $LB^*_{Sup} \equiv (1+\lambda)(C_h^* + t^*) - SB = LB^*_{Inf} + (1+\lambda)\Delta\theta$

The comparative statics of these results are straightforward. On one hand, when the shadow cost of public funds raised by FG, the lowest firm's type, the differential in efficiency and the cost of effort increase, both thresholds also increase. Hence FG funds fewer projects. On the other hand, when the spillover effects are important, more local works are undertaken under full information.

Next assume that, although C is ex-post observable, FG can not distinguish between θ and e. As it has some beliefs about θ, he faces two states of nature $i \in \{l, h\}$, each with a strictly positive probability $p_i \equiv \Pr(\theta = \theta_i)$. FG could induce LG to reveal θ truthfully by offering him an incentive-compatible public works contract. In fact, this is not necessary.

[6] This paper maintains a conventional assumption in contract theory. When F or LG are indifferent between two decisions, they do what FG prefers.

Proposition 2 Under asymmetric information on the efficiency of the firm θ, the optimal allocations that emerge under full information are incentive compatible.

The formal proof appears in Besfamille (1999). Although there is asymmetric information on θ, FG implements the optimal full information allocations with no extra cost by offering, for each state of nature, a public works contract with the corresponding allocation described in Proposition 1. This implies that, for any positive value of the differential in efficiency $\Delta\theta$, at most three possible configurations of decisions concerning the undertaking of the project $(\delta_l; \delta_h)$ can arise:

- [All] : both types of firm undertake the project if $LB \geq LB^*_{Sup}$

θ_l	θ_h
$\delta_l = 1$	$\delta_h = 1$

- [θ_l] : only an efficient firm undertakes the project if $LB \in [LB^*_{Inf}; LB^*_{Sup})$

θ_l	θ_h
$\delta_l = 1$	$\delta_h = 0$

- [None] : if $LB < LB^*_{Inf}$ the project is not undertaken

θ_l	θ_h
$\delta_l = 0$	$\delta_h = 0$

From now on, we call the pair $(\delta_l; \delta_h)$ a "configuration of firms". The first and the last configuration of firms do not discriminate between different types while the second does. These potential configurations of firms coincide with those that would appear under full information. So asymmetric information does not impact in such a way to make new configurations of firms arise. We gather these results in the following figure where, for a given value of θ_l; any point in the graphic represents one project, characterized by the possible values of $\Delta\theta$ and LB: The subscripts indicate that the configurations of firms are implemented by a contract of full information allocations.

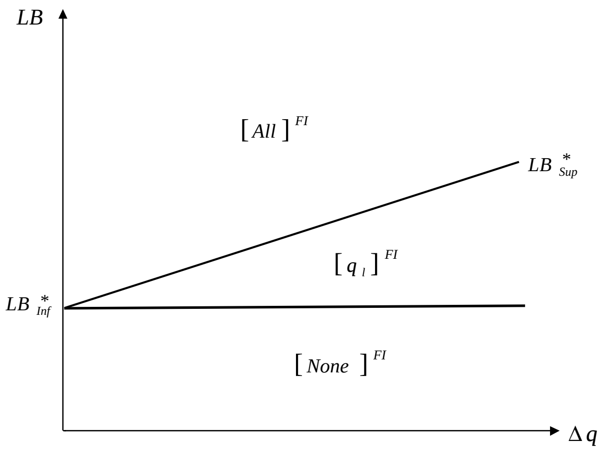

Figure 1: Optimal con¯gurations of ¯rms when corruption is infeasible

4 Corruption

4.1 The covert negotiation: timing and assumptions

Now we relax the assumption that collusion is infeasible. Henceforth side-contracts between the Local Government and the ¯rm of constructors are a possible threat that must be taken in account by the Federal Government at the mechanism-design stage. We characterize the optimal contracts in this new framework.

Prior to the acceptance of the public works contract o®ered by FG, LG might want to coordinate the report to make with F. This covert-negotiation occurs under complete information. Following Tirole (1986), LG o®ers to F a \take-it or leave-it" side-contract. This side-contract, which is supposed to be fully enforceable, speci¯es the ¯nal report that LG should make and the covert-payment b between them. If F rejects this side-contract, LG can only play non-cooperatively the announcement game described in the timing. Therefore, in order to be accepted, the side-contract must be Pareto-superior with respect to this non-cooperative status quo.

4.2 A \Collusion-Proofness" property

As mentioned above, FG must be aware of the threat of collusion between LG and F when it designs the best contract to o®er to LG. The optimization is di±cult because we have not constrained the space of messages M in the public works contract. Fortunately, we can prove the following important result.

Proposition 3 The allocations that maximize the expected welfare of the Federal Government can be implemented by direct-revelation contracts. These contracts are collusion-proof because the Local Government does not gain by coordinating with the firm to deviate from truthful revelation.

FG can restrict himself, without any loss of generality, to offer incentive-compatible collusion-proof contracts to LG. To design those contracts, FG maximizes its expected welfare over the set of contracts that satisfy some collusion-proofness constraints.

4.3 The stakes for collusion: existence

The optimal allocations that emerge under full information can obviously be implemented through direct-revelation mechanisms. But these mechanisms are not robust to collusion. If FG wants to implement the configuration of firms [All] by offering the contract

$$\begin{cases} (\delta = 1; C_l^*; t^*; s^*; \pi) & \text{if } \theta = \theta_l \\ (\delta = 1; C_h^*; t^*; s^*; \pi) & \text{if } \theta = \theta_h \end{cases}$$

the LG and F have incentives to misbehave. When $\theta = \theta_l$, LG can easily convince the firm that the best thing to do is to announce $\tilde{\theta} = \theta_h$. By doing so, they could share an informational rent $\Phi(e^*) = \psi(e^*) - \psi(e^* - \Delta\theta) > 0$.[7] This threat of cost padding always exists.

But in this model another coalitional misbehavior may appear. When FG wants to discriminate between different types of firms to undertake the project, it can offer the contract

$$\begin{cases} (\delta = 1; C_l^*; t^*; s^*; \pi) & \text{if } \theta = \theta_l \\ (\delta = 0; t^o; s^o) & \text{if } \theta = \theta_h \end{cases}$$

When $\theta = \theta_h$, LG might have an incentive to exaggerate the efficiency of the firm by announcing $\tilde{\theta} = \theta_l$ in order to obtain the undertaking of the project. Without corruption, this is impossible. The reason is simple: LG can not avoid F's refusal of the cost-transfer scheme proposed by FG. But here LG can induce an inefficient firm to accept the cost-transfer scheme $(C_l^*; t^*)$ designed for an efficient one. To obtain that, LG should commit to compensate the inefficient firm for the extra effort needed to attain the target cost C_l^*. So this stake for collusion exists if and only if

$$LB + v(-\Phi(e^* + \Delta\theta)) > 0$$

The following lemma states that it is a real threat for FG

Lemma 1 For any positive value of the local benefit of the project LB, there always exists a set of values $\left]0; \overline{\Delta\theta}\right[$ such that if the differential in efficiency $\Delta\theta \in \left]0; \overline{\Delta\theta}\right[$, $LB + v(-\Phi(e^* + \Delta\theta)) > 0$.

[7] Because of the initial assumptions on ψ, the function $\Phi(e)$ verifies $\Phi(e) \geq 0$, $\Phi'(e) > 0$ and $\Phi''(e) > 0$.

When FG wants to implement the configuration of firms $[\theta_l]$; there is always room for this stake of collusion if the differential in efficiency $\Delta\theta$ is low enough. In contrast, when this parameter is high, this stake for collusion disappears because it is too expensive for LG to compensate the firm for the extra effort. This organizational misbehavior concerns LG's subvaluating the cost to obtain the undertaking of the project. Although it seems to be a very widespread phenomenon in public investments, it has only been studied in the literature of cost-overruns from a dynamic perspective, with renegotiation and no-commitment as an issue. Here this problem emerges as a consequence of corruption within a federal hierarchy.

5 Optimal contracts under the threat of corruption

5.1 Cost-minimizing collusion-proof contracts

First of all, we find the cost-minimizing collusion-proof contracts that implement each possible configuration of firms. In order to design them, FG must face the following constraints

- $IR(i)$ and $FIR(i)$: the participation constraint for LG and F respectively.
- $CPC(i)$: the collusion-proofness constraints for LG. They are given by the following two expressions. The first collusion-proofness constraint is

$$\begin{aligned}\delta_l LB + v(s_l) \geq\ & \delta_h[LB + v(s_h + U_h + \Phi(e_h) - U_l)] \\ & + (1-\delta_h)v(s_h + U_h - U_l)\end{aligned} \tag{CPC(l)}$$

  This constraint enables FG to deter the coalitional misbehavior implied by the subvaluation of the efficiency of the firm to gain the informational rent $\Phi(e_h)$. By compensating enough LG in state l, up to the maximum bribe that F is willing to pay in order to obtain the extra informational rent, FG can induce truthful revelation.

  The other collusion-proofness constraint is

$$\begin{aligned}\delta_h LB + v(s_h) \geq\ & \delta_l[LB + v(s_l + U_l - \Phi(e_l + \Delta\theta) - U_h)] \\ & + (1-\delta_l)v(s_l + U_l - U_h)\end{aligned} \tag{CPC(h)}$$

  Now, in order to deter the subvaluation of the cost, LG must gain enough in state h in order to not engaging with F in a socially non-desirable project.

Thus, FG solves the problem

$$
\mathcal{P}\quad
\begin{cases}
\displaystyle \operatorname*{Max}_{\delta_i, e_i; U_i, s_i} \sum_i p_i \{\delta_i [NB - (1+\lambda)(\theta_i - e_i + \psi(e_i))] \\
\qquad\qquad - \lambda U_i - d(s_i)\} \\
\text{subject to} \\
IR(i); FIR(i) \\
CPC(i)
\end{cases}
$$

It is straightforward to prove the following lemma

Lemma 2 An optimal contract must verify $\delta_l \geq \delta_h$.

Hence at most the three mentioned configurations of firms can arise at the optimum. Next we show the cost-minimizing collusion-proof contract that implements each possible configuration of firms. When FG wants the shutdown of the project, no matter the efficiency of the firm, it can obtain that by offering a menu of full information allocations because this configuration of firms [None] is collusion-proof per se. But this is not the case for the other two potential configurations of firms $[\theta_l]$ and [All].

When collusion is infeasible, FG does not distort neither the cost of the project nor the utility of the firm. But a priori, this is not optimal any more because both variables enter in the new constraints $CPC(i)$. Thus FG can distort them in order to attenuate the overall costs of implementation. Moreover, as there are two potential misbehaviors, we show that there are also two different types of cost-distortions in equilibrium, that characterize each configuration of firms.[8]

Proposition 4 The cost-minimizing contract that implements the configuration of firms $[All]^{CP}$ is

$$
\begin{cases}
(\delta = 1; C_l^*; t_l; s_l; \pi) & \text{if } \theta = \theta_l \\
(\delta = 1; C_h; t_h; s_h; \pi) & \text{if } \theta = \theta_h
\end{cases}
$$

where

- $s_l > 0$ and $s_h < 0$
- $s_l = s_h + \Phi(e_h)$ and $LB + v(s_h) \geq 0$
- $C_h > C_h^*$; $t_h < t^*$ and $U_h = t_h - \psi(\theta_h - C_h) = 0$
- $U_l = t_l - \psi(\theta_l - C_l^*) \geq 0$

[8] Now the superscripts will indicate that the configurations of firms are implemented by collusion-proof contracts.

When FG implements the configuration of firms $[All]^{CP}$, it must distort upwardly the cost imposed to the inefficient firm. Not surprisingly, this is the same result that we can find in Laffont and Tirole (1991, 1993). The trade-off between rent extraction and efficiency is solved by imposing a cost for the inefficient firm C_h that is higher than the full information cost C_h^*. But in this model, a fraction of this higher cost must be paid by LG. In fact, FG funds the project by a conditional non-lineal matching grant. As usual, there is no "distortion at the top" because the lowest cost is not distorted. But in state l FG should offer to LG a positive compensation s_l to relax the collusion-proofness constraint. When this compensation is relatively low, the firm does not enjoy any rent; all the implementation cost is paid to the LG. But when the deadweight loss associated to s_l increases and thus it arrives to a threshold, FG starts to offer a net transfer to F such that $U_l > 0$.

Proposition 5 The cost-minimizing contract that implements the configuration of firms $[\theta_l]^{CP}$ is

$$\begin{cases} (\delta = 1; C_l; t_l; s_l; \pi) & \text{if } \theta = \theta_l \\ (\delta = 0; t^o; s_h; \pi) & \text{if } \theta = \theta_h \end{cases}$$

where

- $s_l < 0$ and $s_h > 0$
- $v(s_h) = LB + v(s_l - \beta(e_l + \Delta\theta))$
- $C_l < C_l^*$; $t_l > t^*$ and $U_l = t_l - \alpha(\theta_l - C_l) = 0$

When FG faces the threat of cost subvaluation and tries to implement the configuration of firms $[\theta_l]^{CP}$, it must distort downwardly the target cost imposed to the efficient firm. This implies that the firm that will effectively undertake the project should exert a level of effort e_l, higher than the full information level e^*. By doing that, FG increases the side-transfer that LG should pay to an inefficient firm in order to compensate it to mimic an efficient firm and to undertake the project. As before, FG designs for LG a cost-sharing formula when the project should be done and offers a strictly positive compensation scheme in the other case. Moreover, the utility of the firm remains unchanged, at its reservation level.

5.2 The optimal contracts

Once the cost-minimizing collusion-proof contracts are found, it is straightforward to compute the expected welfare of the Federal Government under each configuration of firms and to look for which values of $\Delta\theta$ and LB the Federal Government implements each possible configuration.

Proposition 6 Under asymmetric information on the e±ciency of the ¯rm µ and collusion between the Local Government and the ¯rm of constructors, the optimal con¯gurations of ¯rms are set as in the following graphic.

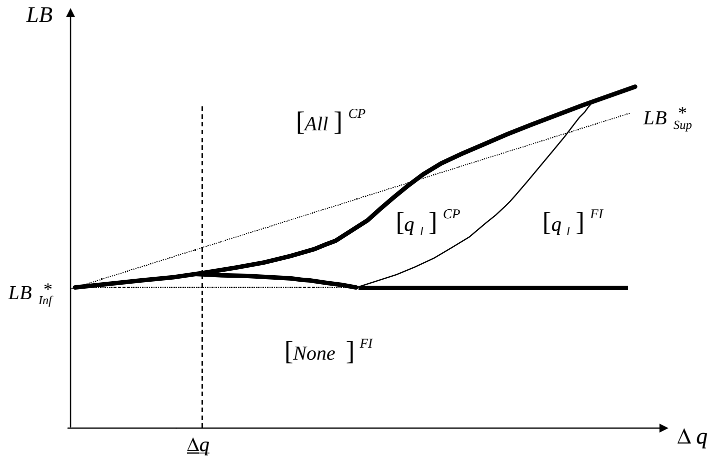

Figure 2: Optimal con¯gurations of ¯rms under the threat of corruption

The graphic shows that, for every con¯guration of ¯rms, there exists a non-empty parametric region where it is optimal to implement it. Collusion is not so constraining so as to make one con¯guration of ¯rms completely disappearing.

Collusion entails upward and downward distortions of the optimal decisions ± about the e®ective undertaking of the project. Upward distortions occur only when, instead of discriminating the ¯rms that will construct the project, FG decides to undertake the work with both types of ¯rms. In the graphic this happens when the con¯guration of ¯rms $[All]^{CP}$ is implemented instead of $[µ_l]^{CP}$. As we have shown in Proposition 4, the contract that implements the undertaking of the project irrespective of the ¯rm's type sets an upwardly distorted target cost for the ine±cient ¯rm. Therefore we obtain the most important result of this paper because we show that the undertaking of useless projects at an in°ated cost is an optimal response to the threat of corruption. Cost overruns for projects that should not be undertaken emerge in a static setting, without any issue concerning the contractual commitment. Downward distortions occur in two cases: when FG decides to shutdown all works instead of discriminating between ¯rms and also when the mentioned discrimination is preferred to the con¯guration of ¯rms $[All]^{CP}$:

These distortions can be very important. For $4µ \quad \underline{4µ}$, the con¯guration of ¯rms $[µ_l]^{CP}$ is no longer optimal and will not be implemented. In that case, when FG wants to discriminate between ¯rms whose e±ciency does not di®er \too much", the stake for collusion is large for LG and F. Therefore, the distortions in the cost of the accepted project in state l and the amount of the transfers to LG needed to attenuate the implementation costs associated to this discrimination are so important that FG shifts towards more drastic distortions in ±.

6 Conclusion

We summarize the main results of this paper. We consider a governmental hierarchy, composed by a Federal Government and a Local Government. The former delegates to the latter the task of ¯nding a ¯rm with the charge of constructing a local public works that generates spillovers. The local authority is informed about the e±ciency of the ¯rm and thus about the cost of the project. It must communicate it to the Federal Government in order to obtain funds for the work. We are able to show, in this setting, the allocative impact of collusion between the Local Government and the ¯rm of constructors.

Our model endogenizes the decision about undertaking the local project. When it should be done independently of its cost, then the usual stake for collusion implying cost-padding arises. But when the project should be undertaken only if it cost is low, the local authority may be tempted to overstate the e±ciency of the ¯rm to obtain the funds to construct it.

Although our model is an incomplete-contract one, we prove a \Collusion-Proofness" property which enables us to easily characterize the optimal allocations. Collusion-proofness yields to distortions concerning the cost of the project and the decision about its e®ective undertaking. When the project should be done, irrespectively of its costs, the latter are distorted upwardly if the ¯rm in charge of the construction is ine±cient. But when the project should be undertaken only if the ¯rm is e±cient, then the target cost is distorted downwardly. Concerning the decisions about the undertaking of the work, we obtain a \two-way" distortion result: more or less projects are done than under full-information. The most important result concerns the emergence of cases where useless projects are optimally undertaken, at an in°ated cost.

These distortions are more important than when collusion between the Local Government and the ¯rm is infeasible, in two aspects. First of all, they appear whereas in a collusion-free setting there are no room for them. Second, these distortions may be so costly that, for a non negligeable region of parameters of the model, the Federal Government abandons the discrimination of projects according to their cost and shifts towards a more drastic way to decide about the funding of local projects. Either they are always undertaken or, on the contrary, always shutdown.

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7 Appendix

7.1 Proof of the "Collusion-Proofness" property

Following Laffont and Tirole (1991 and 1993), this proof is done in different steps. First of all we characterize the equilibrium allocations of overall announcement game. Then we find the allocation that maximizes the expected welfare of FG. Finally we show that this allocation can be implemented through an incentive-compatible collusion-proof contract.

7.1.1 The equilibrium allocations

Any mechanism offered by FG leads to a side-contract between LG and the F and to an equilibrium allocation.[9] We index the final incomes, efforts and utilities by a hat: $\{\hat{t}_i;\hat{s}_i;\hat{e}_i;\hat{U}_i;\hat{V}_i\}_{i\in\{l,h\}}$. The actual transfers from FG to LG and to F are denoted by s_i and t_i. If b_i are the side-transfer between LG and F, we have:

$$\begin{aligned}
\hat{s}_i &= s_i - b_i\\
\hat{t}_i &= t_i + b_i\\
\hat{U}_i &= \hat{t}_i - \delta_i \psi(\hat{e}_i)\\
\hat{V}_i &= \delta_i LB + v(\hat{s}_i)
\end{aligned}$$

Then, in any state of nature, the resulting FG's welfare is

$$\begin{aligned}
\hat{W}_i &\equiv \delta_i[SB - (1+\lambda)C_i] - (1+\lambda)(t_i+s_i) + \hat{U}_i + \hat{V}_i\\
&= \delta_i[NB - (1+\lambda)(C_i + \psi(\hat{e}_i))] - \lambda\hat{U}_i - d(\hat{s}_i)
\end{aligned}$$

We characterize the equilibrium allocations for a particular configuration of firms; the other two are strictly equivalent. Assume that FG wants to implement the configuration of firms [All], where $\delta_i = 1\ \forall i \in \{l,h\}$. The necessary conditions for an allocation to be an equilibrium are the following:

- State l : $\hat{U}_l \geq 0$, $\hat{V}_l \geq 0$ and one of the following possibilities concerning the coalition incentive-compatibility constraints
  - Case l_1

$$\begin{cases}
\hat{U}_h + \Phi(\hat{e}_h) \geq \hat{U}_l\\
v(\hat{s}_l) \geq v(\hat{s}_h + \hat{U}_h + \Phi(\hat{e}_h) - \hat{U}_l)
\end{cases}$$

    F wants to obtain the cost-transfer scheme designed for an inefficient firm in state h but LG does not want that, even if the former gives to the latter all benefits from the desired deviation.

[9] This also embraces the possibility of a non-cooperative outcome.

– Case l_2

$$\begin{cases} \hat{U}_h + \psi(\hat{e}_h) < \hat{U}_l \\ v(\hat{s}_h) \geq v(\hat{s}_l) \\ v(\hat{s}_l) \geq v(\hat{s}_h + \hat{U}_h + \psi(\hat{e}_h) - \hat{U}_l) \end{cases}$$

Although LG wants to deviate, it is too expensive for it to obtain F's acceptation.

- State h : $\hat{U}_h \geq 0$; $\hat{V}_h \geq 0$ and one of the following possibilities, whose intuition are equivalent to the previous ones

– Case h_1

$$\begin{cases} \hat{U}_l - \psi(\hat{e}_l + \Delta\theta) \geq \hat{U}_h \\ v(\hat{s}_h) \geq v(\hat{s}_l + \hat{U}_l - \psi(\hat{e}_l + \Delta\theta) - \hat{U}_h) \end{cases}$$

– Case h_2

$$\begin{cases} \hat{U}_l - \psi(\hat{e}_l + \Delta\theta) < \hat{U}_h \\ v(\hat{s}_l) \geq v(\hat{s}_h) \\ v(\hat{s}_h) \geq v(\hat{s}_l + \hat{U}_l - \psi(\hat{e}_l + \Delta\theta) - \hat{U}_h) \end{cases}$$

Any equilibrium allocation is characterized by a combination of one case for each state of nature. Hence there are four possible cases of equilibrium outcomes, denoted by $(l_1; h_1)$; $(l_1; h_2)$; $(l_2; h_1)$ and $(l_2; h_2)$.

7.1.2 The allocation that maximizes FG's expected welfare for each case

As the aim of FG is to obtain the optimal equilibrium allocation, first we find, in each one of the possible cases mentioned above, the allocation that maximizes FG's expected welfare $E\hat{W}$.

Case $(l_1; h_1)$: FG solves the program

$$
\begin{cases}
\displaystyle\max_{\substack{\hat e_l;\hat U_l;\hat s_l \\ \hat e_h;\hat U_h;\hat s_h}} E\hat W \\
\text{subject to} \\
\hat U_l \geq 0 & (1a) \\
\hat U_h \geq 0 & (1b) \\
LB + v(\hat s_l) \geq 0 & (1c) \\
LB + v(\hat s_h) \geq 0 & (1d) \\
\hat U_h + \phi(\hat e_h) \geq \hat U_l & (1e) \\
\hat U_l - \phi(\hat e_l + \Delta\theta) \geq \hat U_h & (1f) \\
v(\hat s_l) \geq v(\hat s_h + \hat U_h + \phi(\hat e_h) - \hat U_l) & (1g) \\
v(\hat s_h) \geq v(\hat s_l + \hat U_l - \phi(\hat e_l + \Delta\theta) - \hat U_h) & (1h)
\end{cases}
$$

If $(1e); (1f); (1g)$ and $(1h)$ hold, then $\hat s_l = \hat s_h = 0$. So $(1e)$ and $(1f)$ hold with equality, implying $\hat e_l + \Delta\theta = \hat e_h$ and therefore $\hat e_l < \hat e_h$. Moreover, $\hat U_l = \phi(\hat e_l + \Delta\theta) > 0$ and $\hat U_h = 0$. The Lagrangian of the reduced problem is:

$$
\begin{aligned}
L = \; & -p_l[(1+\lambda)(\theta_l - \hat e_l + \psi(\hat e_l)) + \lambda\phi(\hat e_l + \Delta\theta)] \\
& - p_h(1+\lambda)(\theta_h - \hat e_h + \psi(\hat e_h)) + \gamma[\hat e_l + \Delta\theta - \hat e_h]
\end{aligned}
$$

where γ is the multiplier associated with the equality constraint. The first-order conditions yield to

$$
\psi'(\hat e_l) = \frac{1+\lambda}{p_l} - \frac{p_h+\lambda}{p_l}\psi'(\hat e_h)
$$

As $\hat e_l < \hat e_h$, it is straightforward to see that $\hat e_l < e^* < \hat e_h$. With respect to the full information allocation in the configuration of firms $[All]^{FI}$, this equilibrium allocation generates an extra cost

$$
\begin{aligned}
C_1 = \; & p_l[(1+\lambda)(H(\hat e_l) - H(e^*)) + \lambda\phi(\hat e_l + \Delta\theta)] \\
& + p_h(1+\lambda)(H(\hat e_h) - H(e^*))
\end{aligned}
$$

where the function $H(e) \equiv \psi(e) - e$.

Case $(l_1; h_2)$ FG solves the program

$$
P_a \left\{
\begin{array}{ll}
\underset{\substack{\widehat{e}_l;\widehat{U}_l;\widehat{s}_l \\ \widehat{e}_h;\widehat{U}_h;\widehat{s}_h}}{\operatorname{Max}} \ \mathbb{E}\widehat{W} & \\
\text{subject to} & \\
\widehat{U}_l \geq 0 & \text{(2a)} \\
\widehat{U}_h \geq 0 & \text{(2b)} \\
LB + v(\widehat{s}_l) \geq 0 & \text{(2c)} \\
LB + v(\widehat{s}_h) \geq 0 & \text{(2d)} \\
\widehat{U}_h + \Phi(\widehat{e}_h) \geq \widehat{U}_l & \text{(2e)} \\
\widehat{U}_l - \Phi(\widehat{e}_l + \Delta\theta) \quad \widehat{U}_h & \text{(2f)} \\
v(\widehat{s}_l) \geq v(\widehat{s}_h + \widehat{U}_h + \Phi(\widehat{e}_h) - \widehat{U}_l) & \text{(2g)} \\
v(\widehat{s}_h) \geq v(\widehat{s}_l + \widehat{U}_l - \Phi(\widehat{e}_l + \Delta\theta) - \widehat{U}_h) & \text{(2h)}
\end{array}
\right.
$$

If (2d); (2e) and (2g) hold, (2c) also holds. If (2g) and (2h) are verified, $\Phi(\widehat{e}_l + \Delta\theta) \geq \Phi(\widehat{e}_h)$ because the function v is monotonic. Therefore, this last inequality combined with (2e) yields to (2f). The Kuhn-Tucker conditions of the reduced problem are

$$
\left\{
\begin{array}{ll}
-p_l(1+\lambda)(\Psi'(\widehat{e}_l) - 1) + \gamma_6 v'(\widehat{s}_l + \beta)\Phi'(\widehat{e}_l + \Delta\theta) = 0 & \text{(i1)} \\
-p_h(1+\lambda)(\Psi'(\widehat{e}_h) - 1) + \gamma_4\Phi'(\widehat{e}_h) - \gamma_5 v'(\widehat{s}_h + \alpha)\Phi'(\widehat{e}_h) = 0 & \text{(i2)} \\
-\lambda p_l + \gamma_1 - \gamma_4 + \gamma_5 v'(\widehat{s}_h + \alpha) - \gamma_6 v'(\widehat{s}_l + \beta) = 0 & \text{(i3)} \\
-\lambda p_h + \gamma_2 + \gamma_4 - \gamma_5 v'(\widehat{s}_h + \alpha) + \gamma_6 v'(\widehat{s}_l + \beta) = 0 & \text{(i4)} \\
-p_l d'(\widehat{s}_l) + \gamma_5 v'(\widehat{s}_l) - \gamma_6 v'(\widehat{s}_l + \beta) = 0 & \text{(i5)} \\
-p_h d'(\widehat{s}_h) + \gamma_3 v'(\widehat{s}_h) - \gamma_5 v'(\widehat{s}_h + \alpha) + \gamma_6 v'(\widehat{s}_h) = 0 & \text{(i6)} \\
\gamma_1 \widehat{U}_l = 0 \qquad \gamma_1 \geq 0 & \text{(ii1)} \\
\gamma_2 \widehat{U}_h = 0 \qquad \gamma_2 \geq 0 & \text{(ii2)} \\
\gamma_3[LB + v(\widehat{s}_h)] = 0 \qquad \gamma_3 \geq 0 & \text{(ii3)} \\
\gamma_4[\widehat{U}_h + \Phi(\widehat{e}_h) - \widehat{U}_l] = 0 \qquad \gamma_4 \geq 0 & \text{(ii4)} \\
\gamma_5[v(\widehat{s}_l) - v(\widehat{s}_h + \alpha)] = 0 \qquad \gamma_5 \geq 0 & \text{(ii5)} \\
\gamma_6[v(\widehat{s}_h) - v(\widehat{s}_l + \beta)] = 0 \qquad \gamma_6 \geq 0 & \text{(ii6)}
\end{array}
\right.
$$

where $\gamma_j \ \forall j \in \{1;2;3;4;5;6\}$ are the multipliers associated with the inequality constraints, $\alpha \equiv \widehat{U}_h + \Phi(\widehat{e}_h) - \widehat{U}_l \geq 0$ and $\beta \equiv \widehat{U}_l - \Phi(\widehat{e}_l + \Delta\theta) - \widehat{U}_h \quad 0$:

1. From (i3) and (i4); $\gamma_1 + \gamma_2 = \lambda$:
2. Then we show that $\gamma_6 = 0$: Assume $\gamma_6 > 0$: Hence, from (ii6); $\widehat{s}_h = \widehat{s}_l + \beta$ and from (i1); $\widehat{e}_l > e^*$: But if (ii5) holds, it yields to $\widehat{e}_h \quad \widehat{e}_l + \Delta\theta$:

(a) If $\widehat{e}_h = \widehat{e}_l + \triangle\theta; \widehat{e}_h > e^*$: But from (i2); this should imply that $\gamma_4 - \gamma_5 v'(\widehat{s}_h + \alpha) > 0$ and therefore $\gamma_4 > 0$: If so, $\alpha = 0$ and $\widehat{U}_l > 0$; which then implies that $\gamma_1 = 0$: But (i3) becomes

$$0 > -\lambda p_l - \gamma_6 v'(\widehat{s}_l + \beta) = \gamma_4 - \gamma_5 v'(\widehat{s}_h) > 0$$

which is a contradiction.

(b) If $\widehat{e}_h < \widehat{e}_l + \triangle\theta; \alpha + \beta < 0$ and so (ii5) is slack, implying $\gamma_5 = 0$: If this was the case, (i2) yields to

$$\widehat{e}_h \geq e^*$$

(i5) yields to $\widehat{s}_l < 0$ and (i6) yields to $\widehat{s}_h \geq 0$: Hence, as $\beta \quad 0$

$$\widehat{s}_l < 0 \qquad \widehat{s}_h = \widehat{s}_l + \beta \qquad \widehat{s}_l$$

which is another contradiction.

So $\gamma_6 = 0$ and $\widehat{e}_l = e^*$: Moreover, (i5) becomes

$$p_l d'(\widehat{s}_l) = \gamma_5 v'(\widehat{s}_l) \tag{1}$$

so, as $\gamma_5 \geq 0; \widehat{s}_l \geq 0$:

3. Next we prove that $\widehat{U}_h + \Phi(\widehat{e}_h) > \widehat{U}_l$: Assume that $\widehat{U}_h + \Phi(\widehat{e}_h) = \widehat{U}_l$ so $\alpha = 0$: As $\Phi(\widehat{e}_h) > 0; \widehat{U}_l > 0$ and so $\gamma_1 = 0$: In that case, (i3) yields to $\gamma_5 > 0$ and $\widehat{s}_h = \widehat{s}_l$: This equality, combined with (i6); yields to

$$\gamma_3 v'(\widehat{s}_h) = d'(\widehat{s}_h)$$

which implies that $\widehat{s}_h \geq 0$ and therefore $\gamma_3 = 0$ because (ii3) is slack. If so, $\widehat{s}_h = 0$: But then, $\widehat{s}_l = 0$ which must imply that $\gamma_5 = 0$ in order to satisfy (1). But this is a contradiction.

So $\alpha \equiv \widehat{U}_h + \Phi(\widehat{e}_h) - \widehat{U}_l > 0$ and therefore $\gamma_4 = 0$: If so, (i4) becomes

$$\gamma_2 = \lambda p_h + \gamma_5 v'(\widehat{s}_h + \alpha) > 0$$

so $\widehat{U}_h = 0$:

4. Next we prove that $\widehat{s}_l > 0$: Assume $\widehat{s}_l = 0$: Hence $\gamma_5 = 0$ and (i6) becomes

$$\gamma_3 v'(\widehat{s}_h) = d'(\widehat{s}_h)$$

which yields to $\widehat{s}_h \geq 0$: But from (ii5) and the fact that $\alpha > 0$

$$0 = \widehat{s}_l \geq \widehat{s}_h + \alpha > 0$$

which is obviously a contradiction.

Hence $\widehat{s}_l > 0$ so, from (1), $\gamma_5 > 0$ and therefore $\widehat{s}_l = \widehat{s}_h + \alpha$ and $\widehat{e}_h < e^*$ from (i2):

5. Next we show for each values of the parameters 4μ and LB the participation constraints (2a) and (2d) bind: Using the Kuhn-Tucker conditions, it is straightforward to see the result in the following graphic, where the frontiers are drawn applying the Implicit Function Theorem.

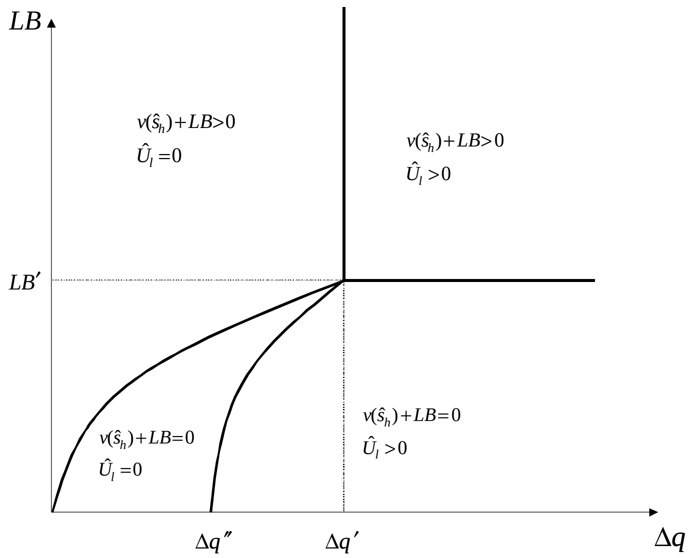

Figure 3

The particular values depicted there are characterized as follows. Let $\hat{s}_h$ be de¯ned by

$$v^0(\hat{s}_h) = 1 + \frac{¸}{p_h}$$

$\hat{s}_l$ be de¯ned by

$$v^0(\hat{s}_l) = 1$$

and LB^0 by

$$v(\hat{s}_h) + LB^0 = 0$$

When (2a) is slack, $\hat{b}_h$ is characterized by the following expression

$$ª^0(\hat{b}_h) = 1 ¡ \frac{p_l}{p_h}\frac{¸}{1+¸}©^0(\hat{b}_h)$$

so $4\mu^0$ is de¯ned by

$$©(\hat{b}_h(4\mu^0)) = \hat{s}_l ¡ \hat{s}_h$$

and $4\mu^{00}$ by

$$©(\hat{b}_h(4\mu^{00})) = \hat{s}_l$$

This allocation generates an extra cost

$$
C_2 = \begin{cases} p_l d(\hat{s}_l) + p_h\left[(1+\lambda)(H(\hat{e}_h) - H(e^*)) + d(\hat{s}_h)\right] & \text{if (2a) binds} \\ p_l\left[d(\hat{s}_l) + \lambda \hat{U}_l\right] + p_h\left[(1+\lambda)(H(\hat{e}_h) - H(e^*)) + d(\hat{s}_h)\right] & \text{if (2a) is slack} \end{cases}
$$

Case (l_2, h_1) FG solves the program

$$
\begin{cases}
\displaystyle \operatorname*{Max}_{\substack{\hat{e}_l, \hat{U}_l, \hat{s}_l \\ \hat{e}_h, \hat{U}_h, \hat{s}_h}} \widehat{EW} \\
\text{subject to} \\
\begin{cases}
\hat{U}_l \geq 0 & \text{(3a)} \\
\hat{U}_h \geq 0 & \text{(3b)} \\
LB + v(\hat{s}_l) \geq 0 & \text{(3c)} \\
LB + v(\hat{s}_h) \geq 0 & \text{(3d)} \\
\hat{U}_h + \Phi(\hat{e}_h) \leq \hat{U}_l & \text{(3e)} \\
\hat{U}_l - \Phi(\hat{e}_l + \Delta\theta) \geq \hat{U}_h & \text{(3f)} \\
v(\hat{s}_h) \geq v(\hat{s}_l) \geq v(\hat{s}_h + \hat{U}_h + \Phi(\hat{e}_h) - \hat{U}_l) & \text{(3g)} \\
v(\hat{s}_h) \geq v(\hat{s}_l + \hat{U}_l - \Phi(\hat{e}_l + \Delta\theta) - \hat{U}_h) & \text{(3h)}
\end{cases}
\end{cases}
$$

If (3b) and (3e) hold, then (3a) also holds. If (3f) and (3h) hold, (3g) also holds. If (3c) and (3h) hold, then (3d) is satisfied. So the Kuhn-Tucker conditions of the reduced problem are

$$
\begin{cases}
\begin{cases}
-p_l(1+\lambda)(\Psi'(\hat{e}_l) - 1) + [\gamma_6 v'(\hat{s}_l + \beta) - \gamma_4]\Phi'(\hat{e}_l + \Delta\theta) = 0 & \text{(i1)} \\
-p_h(1+\lambda)(\Psi'(\hat{e}_h) - 1) - [\gamma_3 + \gamma_5 v'(\hat{s}_h + \alpha)]\Phi'(\hat{e}_h) = 0 & \text{(i2)} \\
-\lambda p_l + \gamma_3 + \gamma_4 + \gamma_5 v'(\hat{s}_h + \alpha) - \gamma_6 v'(\hat{s}_l + \beta) = 0 & \text{(i3)} \\
-\lambda p_h + \gamma_1 - \gamma_3 - \gamma_4 - \gamma_5 v'(\hat{s}_h + \alpha) + \gamma_6 v'(\hat{s}_l + \beta) = 0 & \text{(i4)} \\
-p_l d'(\hat{s}_l) + \gamma_2 v'(\hat{s}_l) + \gamma_5 v'(\hat{s}_l) - \gamma_6 v'(\hat{s}_l + \beta) = 0 & \text{(i5)} \\
-p_h d'(\hat{s}_h) - \gamma_5 v'(\hat{s}_h + \alpha) + \gamma_6 v'(\hat{s}_h) = 0 & \text{(i6)}
\end{cases} \\
\begin{cases}
\gamma_1 \hat{U}_h = 0 \qquad \gamma_1 \geq 0 & \text{(ii1)} \\
\gamma_2[LB + v(\hat{s}_h)] = 0 \qquad \gamma_2 \geq 0 & \text{(ii2)} \\
\gamma_3[\hat{U}_l - \hat{U}_h - \Phi(\hat{e}_h)] = 0 \qquad \gamma_3 \geq 0 & \text{(ii3)} \\
\gamma_4[\hat{U}_l - \hat{U}_h - \Phi(\hat{e}_l + \Delta\theta)] = 0 \qquad \gamma_4 \geq 0 & \text{(ii4)} \\
\gamma_5[v(\hat{s}_l) - v(\hat{s}_h + \alpha)] = 0 \qquad \gamma_5 \geq 0 & \text{(ii5)} \\
\gamma_6[v(\hat{s}_h) - v(\hat{s}_l + \beta)] = 0 \qquad \gamma_6 \geq 0 & \text{(ii6)}
\end{cases}
\end{cases}
$$

where $\alpha \leq 0$ and $\beta \geq 0$. For the moment, we assume that (ii2) is slack. Then we have to verify this statement.

1. From (i3) and (i4), $\gamma_1 = \lambda > 0$. Hence $\hat{U}_h = 0$.

2. Next we show that at the maximum $\hat{\tau}_l = \varphi(\tau_l + 4\theta)$. If we assume that $\hat{\tau}_l > \varphi(\tau_l + 4\theta)$, we must have $\gamma_4 = 0$. We show that this statement yields to a global contradiction. We can have either $\gamma_6 = 0$ or $\gamma_6 > 0$.

   • If $\gamma_6 = 0$, from (i1), $\tau_l = e^*$ and from (i4)

   $$\gamma_3 + \gamma_5 v'(t_h + \alpha) = \lambda p_l > 0 \tag{2}$$

   which implies that $\tau_h < e^*$. So $\varphi(\tau_h) < \varphi(e^*) < \varphi(\tau_l + 4\theta)$. Hence $\hat{\tau}_l > \varphi(\tau_h)$ so $\alpha < 0$ and $\gamma_3 = 0$. If so, (2) imply that $\gamma_5 > 0$ so

   $$t_l = t_h + \alpha \tag{3}$$

   which yields to $t_l < t_h$ because $\alpha < 0$. Plugging (2) in (i6) yields to $t_h < 0$ so, from (3), $t_l < 0$. But also plugging (2) and (3) in (i5) yields to $t_l \geq 0$ which is a contradiction.

   • If $\gamma_6 > 0$, $t_h = t_l + \beta$, $\tau_l > e^*$ and from (i3)

   $$0 < \lambda p_l + \gamma_6 v'(t_l + \beta) = \gamma_3 + \gamma_5 v'(t_h + \alpha)$$

   so $\tau_h < e^*$. Hence $\hat{\tau}_l > \varphi(\tau_h)$ and $\gamma_3 = 0$. If so, $\gamma_5 > 0$ and therefore (2) holds again. But combined with the value of t_h, it follows that $\varphi(\tau_h) = \varphi(\tau_l + 4\theta)$ which is a contradiction.

   So $\hat{\tau}_l = \varphi(\tau_l + 4\theta)$ and therefore $\beta = 0$. As (ii5) and (ii6) hold, $v(t_h) \geq v(t_l) \geq v(t_h + \alpha)$.

3. Next we show that $t_h = t_l$. Assume $t_h > t_l$ so $\gamma_6 = 0$. (i5) yields to $t_l \geq 0$ while (i6) to $t_h \leq 0$, so by the assumption, $t_l < 0$ which is a contradiction. So as $t_h = t_l$, (i5) becomes

   $$\gamma_6 - \gamma_5 = \frac{-p_l d'(t_l)}{v'(t_l)} \tag{4}$$

   and, as $\alpha \leq 0$, (i6) becomes

   $$p_h d'(t_h) = \gamma_6 v'(t_h) - \gamma_5 v'(t_h + \alpha) \leq (\gamma_6 - \gamma_5) v'(t_h)$$

   Combining the equality between the transfers to LG and (4), we obtain $t_h = t_l \leq 0$.

4. Next we show that, at the maximum, $\varphi(\tau_l + 4\theta) > \varphi(\tau_h)$. Assume they are equal. Hence $\alpha = 0$ and $\tau_l < \tau_l + 4\theta = \tau_h \leq e^*$ so

   $$\tau_l < e^* \tag{5}$$

(i6) yields to

$$\gamma_6 - \gamma_5 = \frac{p_l d'(\hat{e}_h)}{v'(\hat{e}_h)}$$

Combining this last expression with (4) yields to $\hat{e}_h = \hat{e}_l = 0$ and hence $\gamma_5 = \gamma_6$. Therefore (i3) becomes $\gamma_3 = \lambda p_l - \gamma_4$. We thus obtain the following system

$$\begin{cases} \Psi'(\hat{e}_l) = 1 + \frac{1}{p_l(1+\lambda)}[\gamma_6(1+\lambda) - \gamma_4]\Phi'(\hat{e}_h) \\ \Psi'(\hat{e}_h) = 1 - \frac{1}{p_h(1+\lambda)}[\gamma_3 + \gamma_6(1+\lambda)]\Phi'(\hat{e}_h) \\ \gamma_3 = \lambda p_l - \gamma_4 \end{cases}$$

From the first equation, we can obtain

$$\gamma_6 \Phi'(\hat{e}_h) = p_l[\Psi'(\hat{e}_l) - 1] + \frac{\gamma_4}{(1+\lambda)}\Phi'(\hat{e}_h)$$

If we plug it in the second equation, we have

$$\Psi'(\hat{e}_h) = \frac{1+\lambda}{p_h+\lambda} - \frac{p_l}{p_h+\lambda}\Psi'(\hat{e}_l)$$

But from (5), $\Psi'(\hat{e}_l) < 1$ so $\Psi'(\hat{e}_h) > 1$ which is a contradiction. Hence $\Phi(\hat{e}_l + \Delta\theta) > \Phi(\hat{e}_h)$.

5. As a consequence of the previous result, $\alpha < 0$ so $\gamma_5 = 0$ and (ii3) is slack so also $\gamma_3 = 0$. But then $\gamma_3 = \gamma_5 = 0$ so $\hat{e}_h = e^*$ and from (i3), $\gamma_4 - \gamma_6 v'(\hat{e}_l) = \lambda p_l > 0$ so $\hat{e}_l < e^*$.

6. Finally, from (i6),

$$\gamma_6 = p_h\left(\frac{1+\lambda}{v'(\hat{e}_h)} - 1\right)$$

which yields to $\gamma_6 = 0$ as the only compatible solution. So $\hat{e}_l = \hat{e}_h = 0$ and $\gamma_4 = \lambda p_l$.

This allocation generates a cost of implementation

$$C_3 = p_l[(1+\lambda)(H(\hat{e}_l) - H(e^*)) + \lambda\Phi(\hat{e}_l + \Delta\theta))]$$

Case $(l_2; h_2)$ FG solves the program

$$
\begin{cases}
\underset{\substack{\hat e_l;\hat u_l;\hat s_l\\ \hat e_h;\hat u_h;\hat s_h}}{\text{Max}} \ \mathbb{E}\widehat{W} \\
\text{subject to} \\
\hat u_l \ge 0 & \text{(4a)}\\
\hat u_h \ge 0 & \text{(4b)}\\
LB + v(\hat s_l) \ge 0 & \text{(4c)}\\
LB + v(\hat s_h) \ge 0 & \text{(4d)}\\
\hat u_h + \psi(\hat e_h) \le \hat u_l & \text{(4e)}\\
\hat u_l - \psi(\hat e_l + \Delta\theta) \le \hat u_h & \text{(4f)}\\
v(\hat s_h) \ge v(\hat s_l) \ge v(\hat s_h + \hat u_h + \psi(\hat e_h) - \hat u_l) & \text{(4g)}\\
v(\hat s_l) \ge v(\hat s_h) \ge v(\hat s_l + \hat u_l - \psi(\hat e_l + \Delta\theta) - \hat u_h) & \text{(4h)}
\end{cases}
$$

If (4b) and (4e) hold, then (4a) is satisfied. If (4g) and (4h) hold, then $\hat s_l = \hat s_h = 0$. So if (4d) holds, then (4c) also holds. So (4e) and (4f) hold with equality, implying that $\hat e_l + \Delta\theta = \hat e_h$ and therefore $\hat e_l < \hat e_h$. Moreover, $\hat u_l = \psi(\hat e_l + \Delta\theta) > 0$ and $\hat u_h = 0$. This case is formally identical with the first case $(l_1; h_1)$ so its implementation cost verifies $C_4 = C_1$.

7.1.3 The optimal allocation

Although FG can implement the configuration of firms [All] in four different ways, it will do it with the one that minimizes the extra cost. So we compare them: As for each value of $\Delta\theta$ there exist a solution for the different programs, we can show that

1. $\lim_{\Delta\theta \to 0^+} C_1 = \lim_{\Delta\theta \to 0^+} C_2 = \lim_{\Delta\theta \to 0^+} C_3 = 0$

2. Next we apply the Envelope Theorem to compute the derivatives of the different costs with respect to $\Delta\theta$.

   - As $\hat e_h = \hat e_l + \Delta\theta$ in case $(l_1; h_1)$; $\frac{de_h}{d\Delta\theta} = \frac{de_l}{d\Delta\theta} + 1$ so

   $$\frac{dC_1}{d\Delta\theta} = \frac{dC_4}{d\Delta\theta} = (p_h + \lambda)(\varphi'(\hat e_h) - 1) + \lambda p_l > \lambda p_l$$

   because, as $\hat e_h > e^*$, $\varphi'(\hat e_h) > 1$.

- In case $(l_1; h_2)$ $\hat{s}_l = \hat{s}_h + \Phi(\hat{e}_h)$. So $\frac{d\hat{s}_l}{d\Delta\theta} = \frac{d\hat{s}_h}{d\Delta\theta} + \Phi'(\hat{e}_h)\frac{de_h}{d\Delta\theta} + \Psi'(\hat{e}_h - \Delta\theta)$ and therefore

$$\begin{aligned} \frac{dC_2}{d\Delta\theta} &= p_l d'(\hat{s}_l)(\Psi'(\hat{e}_h - \Delta\theta) & \text{if } \hat{U}_l = 0 \\ &= p_l \lambda \Psi'(\hat{e}_h - \Delta\theta) & \text{if } \hat{U}_l > 0 \end{aligned}$$

  As $\Psi'(\hat{e}_h - \Delta\theta) < 1$ and $v'(\hat{s}_l) \geq 1$

$$\frac{dC_2}{d\Delta\theta} < \lambda p_l$$

- As $\hat{e}_l + \Delta\theta > e^*$ in case $(l_2; h_1)$;

$$\frac{dC_3}{d\Delta\theta} = \lambda p_l \Psi'(\hat{e}_l + \Delta\theta) > \lambda p_l$$

The final result is immediate. FG implements the configuration of firms [All] by offering a contract that yields to the second case of equilibrium allocation because this case minimizes the implementation costs. It is straightforward to verify that this allocation can also be implemented by a direct-revelation collusion-proof public works contract, with no bribes in equilibrium ■

7.2 Proof of Lemma 1

Take on arbitrary combination of parameters and functions $\Lambda = (SB; \lambda; p_i; \theta_l; \Psi; v)$ of the model. Then take any given $LB > 0$. We try to find if there exist values of $\Delta\theta$ such that

$$LB + v(-\Phi(e^* + \Delta\theta)) \geq 0 \tag{6}$$

We analyze the shape of the function $G_{LB}(\Delta\theta) \equiv LB + v(-\Phi(e^* + \Delta\theta))$:

- $\lim_{\Delta\theta \to 0^+} G_{LB} = LB > 0$
- $\frac{dG_{LB}}{d\Delta\theta} = -v'(-\Phi(e^* + \Delta\theta))\Psi'(e^* + \Delta\theta) < -(1 + \lambda) < 0$
- $\frac{d^2G_{LB}}{d\Delta\theta^2} = v''(-\Phi(e^* + \Delta\theta))\Psi'(e^* + \Delta\theta) - v'(-\Phi(e^* + \Delta\theta)\Psi''(e^* + \Delta\theta) < 0$

So there exists a unique value $\overline{\Delta\theta}_{LB} > 0$ such that $G_{LB}(\overline{\Delta\theta}_{LB}) = 0$. Hence we have found an open non-empty interval $(0; \overline{\Delta\theta}_{LB}]$ where $\forall\Delta\theta \leq \overline{\Delta\theta}_{LB}$; $G_{LB}(\Delta\theta) \geq 0$ ■

7.3 Proof of Lemma 2

Assume that an optimal contract yields to $\delta_l < \delta_h$ (i.e. $\delta_l = 0$ and $\delta_h = 1$). From the collusion-proofness constraints we can state that $v(s_l) > 0$ and $U_h > 0$. Moreover, in order to attenuate the distortions in the transfers, $e_h \leq e^*$. Hence

$$\mathbb{E}W_{[\delta_l=0;\delta_h=1]} = -p_l d(s_l) + p_h\{NB - (1+\lambda)(C_h + \psi(e_h)) - \lambda U_h - d(s_h)\}$$

where, at least, $v(s_l) \geq LB + v(s_h + U_h + \Phi(e_h))$. But if this configuration of firms is implemented, it means that $\mathbb{E}W_{[\delta_l=0;\delta_h=1]} \geq 0$ or equivalently that $NB - (1+\lambda)(C_h + \psi(e_h)) - \lambda U_h - d(s_h) > 0$. But as $NB - (1+\lambda)(C_h + \psi(e_h)) - \lambda U_h - d(s_h) < NB - (1+\lambda)(C_l^* + \psi(e^*))$, it is worth to undertake the project in the state l. So this configuration of firms is always dominated by another when the project is undertaken in both states of nature and letting $U_l = 0$ and $e_l = e^*$, which is a contradiction ■

7.4 Cost-minimizing collusion-proof contracts

7.4.1 $[All]^{CP}$

The cost-minimizing collusion-proof contract that implements the configuration of firms $[All]^{CP}$ solves the following problem

$$P_1 \begin{cases} \underset{\substack{e_l;U_l;s_l\\ e_h;U_h;s_h}}{\text{Max}} \; p_l\{NB - (1+\lambda)(\theta_l - e_l + \psi(e_l)) - \lambda U_l - d(s_l)\} + \\ \qquad p_h\{NB - (1+\lambda)(\theta_h - e_h + \psi(e_h)) - \lambda U_h - d(s_h)\} \\ \text{subject to} \\ U_l \geq 0 & FIR(l) \\ U_h \geq 0 & FIR(h) \\ LB + v(s_l) \geq 0 & IR(l) \\ LB + v(s_h) \geq 0 & IR(h) \\ v(\hat{s}_l) \geq v(s_h + U_h + \Phi(e_h) - U_l) & CPC(l) \\ v(s_h) \geq v(s_l + U_l - \Phi(e_l + \Delta\theta) - U_h) & CPC(h) \end{cases}$$

Taking into account the fact that the constraint (2e) in the case $(l_1; h_2)$ is always slack, P_1 is formally equivalent than the program P_a. Therefore the solutions are also the same, so $e_l = e^*; e_h = \hat{e}_h; s_l = \hat{s}_l; s_h = \hat{s}_h$ and $U_l = \hat{U}_l$. The values of $\Delta\theta$ and LB for which $FIR(l)$ and $IR(h)$ bind can be seen in Figure 3.

7.4.2 $[\theta_l]^{CP}$

We characterize the cost-minimizing collusion-proof contract that implements the configuration of firms $[\theta_l]^{CP}$ when, by assumption, the stake for collusion is effective. This is true for values that verify

$$LB + v(-\Phi(e^* + \Delta\theta)) > 0 \tag{7}$$

FG must solve the following problem

$$P_2 \begin{cases} \displaystyle\operatorname*{Max}_{\substack{e_l;U_l;s_l\\ s_h}} \; p_l\{NB - (1+\lambda)(\theta_l - e_l + \Psi(e_l)) - \lambda U_l - d(s_l)\} \\ \qquad\qquad - p_h d(s_h) \\ \text{subject to} \\ \begin{cases} U_l \geq 0 & FIR(l) \\ LB + v(s_l) \geq 0 & IR(l) \\ v(s_h) \geq 0 & IR(h) \\ LB + v(s_l) \geq v(s_h - U_l) & CPC(l) \\ v(s_h) \geq LB + v(s_l + U_l - \Phi(e_l + \Delta\theta)) & CPC(h) \end{cases} \end{cases}$$

We forget for the moment $CPC(l)$. Then we check if the solution verifies it. The Kuhn-Tucker conditions of P_2 are

$$\begin{cases} -\lambda p_l + \gamma_1 - \gamma_5 v'(s_l + \beta) = 0 & (i1) \\ -p_l(1+\lambda)(\Psi'(e_l) - 1) + \gamma_4 v'(s_l + \beta)\Phi'(e_l + \Delta\theta) = 0 & (i2) \\ -p_l d'(s_l) + \gamma_2 v'(s_l) - \gamma_4 v'(s_l + \beta) = 0 & (i3) \\ -p_h d'(s_h) + (\gamma_3 + \gamma_4) v'(s_h) = 0 & (i4) \end{cases}$$

$$\begin{cases} \gamma_1 U_l = 0 \qquad \gamma_1 \geq 0 & (ii1) \\ \gamma_2[LB + v(s_l)] = 0 \qquad \gamma_2 \geq 0 & (ii2) \\ \gamma_3 v(s_h) = 0 \qquad \gamma_3 \geq 0 & (ii3) \\ \gamma_4[v(s_h) - LB - v(s_l + \beta)] = 0 \qquad \gamma_4 \geq 0 & (ii4) \end{cases}$$

where $\beta \equiv U_l - \Phi(e_l + \Delta\theta)$.

1. By simple observation of (i1), $\gamma_1 = \lambda p_l + \gamma_4 v'(s_l + \beta) > 0$. Hence $U_l = 0$ and $b < 0$.

2. From (i2),

$$\Psi'(e_l) = 1 + \gamma_4 \frac{1}{p_l} \frac{v'(s_l + \beta)}{1+\lambda} \Phi'(e_l + \Delta\theta) \geq 1$$

   which implies that $e_l \geq e^*$.

3. Next we claim that $v(s_h) = LB + v(s_l + \beta)$. Assume that $v(s_h) > LB + v(s_l + \beta)$ so $\gamma_4 = 0$. This has the following consequences:

   - from (i2), $e_l = e^*$

- (i3) becomes
$$\gamma_2 v'(s_l) = p_l d'(s_l) \geq 0 \tag{8}$$
In order to satisfy it, $s_l \geq 0$: Hence (ii2) is slack and then $\gamma_2 = 0$. Therefore, to verify (8); $s_l = 0$:
- (i4) becomes
$$\gamma_3 v'(s_h) = p_h(1 + \lambda - v'(s_h)) \tag{9}$$

So, from the initial assumption about the stake for collusion and the statement at the beginning of this point, we have that
$$v(s_h) > LB + v(-\Phi(e^* + \Delta\theta)) > 0$$
which implies that $v(s_h) > 0$ and so $\gamma_3 = 0$: But in that case, the only way to satisfy (9) is by $s_h = 0$; which is a contradiction. Hence $v(s_h) = LB + v(s_l + \beta)$.

4. As $\beta < 0$ and (ii3) must hold,
$$LB + v(s_l) > LB + v(s_l + \beta) = v(s_h) \geq 0$$
so (ii2) is slack and $\gamma_2 = 0$: Moreover, $CPC(l)$ is effectively slack.

5. Next we claim that $\gamma_4 > 0$: Assume that $\gamma_4 = 0$; which has the following consequences

   - from (i2); $e_l = e^*$
   - (i3) becomes
   $$p_l d'(s_l) = 0$$
   which implies that $s_l = 0$
   - (i4) becomes
   $$\gamma_3 v'(s_h) = p_h(1 + \lambda - v'(s_h)) \tag{10}$$

   But we have already proved that $v(s_h) = LB + v(s_l + \beta)$ so $v(s_h) = LB + v(-\Phi(e^* + \Delta\theta)) > 0$ from the initial assumption. So $s_h > 0$ and therefore $\gamma_3 = 0$ if (ii4) has to be verified. But then, the only way to verify (10) is $s_h = 0$, which is a contradiction. Hence $\gamma_4 > 0$ and so $e_l > e^*$.

6. (i3) becomes
$$-p_l d'(s_l) = \gamma_4 v'(s_l + \beta) > 0$$
so $s_l < 0$.

7. Next we claim that $s_h > 0$: Assume that $s_h = 0$ so $v'(s_h) = 1 + \lambda$: (i4) becomes
$$(\gamma_3 + \gamma_4)(1 + \lambda) = 0$$
which is a contradiction because we have already proved that $\gamma_4 > 0$: Hence $s_h > 0$ and then $\gamma_3 = 0$ ■

7.5 Proof of Proposition 6

In order to draw the frontiers of the parametric regions where FG optimally implements each configuration of firms by offering the contracts characterized above, we proceed by comparing the different values of the expected welfare in each case.

1. First of all, we compare $\mathbb{E}W_{[All]^{CP}}$ and $\mathbb{E}W_{[\theta_l]^{CP}}$. Let's compute

$$\left.\frac{d\mathbb{E}W_{[All]^{CP}}}{d\Delta\theta}\right|_{LB=LB^*_{Sup}} = (1+\lambda) - p_l d'(s_l)\, a'(e_h - \Delta\theta)$$

and

$$\left.\frac{d\mathbb{E}_{[\theta_l]^{CP}}}{d\Delta\theta}\right|_{LB=LB^*_{Sup}} = p_l(1+\lambda) + p_h d'(s_h)\left[\frac{v'(s_l+\beta)}{v'(s_h)}\, a'(e_l+\Delta\theta) - (1+\lambda)\right]$$

when $v(s_h) + LB > 0$ and $U_l = 0$:

Then take a sequence of pairs $\{(\Delta\theta)_n, (LB)_n\}$ verifying

$$\begin{cases} \lim_{n\to\infty} (\Delta\theta)_n = 0 \\ (LB)_n = LB^*_{Sup} = LB^*_{Inf} + (1+\lambda)(\Delta\theta)_n \end{cases}$$

We know that

$$\lim_{n\to\infty} \mathbb{E}W_{[All]^{CP}} = \lim_{n\to\infty} \mathbb{E}W_{[\theta_l]^{CP}}$$

Taking limits when $n\to\infty$; we obtain

$$\left.\frac{d\mathbb{E}W_{[All]^{CP}}}{d\Delta\theta}\right|_{LB=LB^*_{Sup}} = (1+\lambda)$$

and

$$\left.\frac{d\mathbb{E}W_{[\theta_l]^{CP}}}{d\Delta\theta}\right|_{LB=LB^*_{Sup}} = p_l(1+\lambda)$$

Therefore, starting from $(\Delta\theta; LB) = \left(0; LB^*_{Inf}\right)$; when the differential in efficiency $\Delta\theta$ increases and $LB = LB^*_{Sup}$, FG strictly prefers to implement the configuration of firms $[All]^{CP}$ instead of $[\theta_l]^{CP}$ whereas, under full information, it was indifferent between them.

Moreover, we showed in Lemma 1 that, for sufficiently high values of $\Delta\theta$; the discrimination between different firms can be implemented by a menu of full information allocations (i.e. without extra costs). Therefore, from a certain value of $\Delta\theta$, the configuration of firms $[\theta_l]^{FI}$ must dominate the other $[All]^{CP}$.

2. The comparison between the con¯guration of ¯rms $[\mu_l]^{CP}$ and [None] is straightforward. Take projects verifying 4μ 2 $(0;\overline{4\mu}]$ and $LB = LB^{¤}_{Inf}$. The implementation of the ¯rst con¯guration of ¯rms yields to strictly positive costs due to collusion-proofness. Therefore the second con¯guration dominates, whereas under full information FG was indi®erent between them ■