On the Nature of Price Competition under Universal Service Obligations: a Note*

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Abstract

The imposition of universal coverage and uniform pricing constraints, as part of the universal service obligations, makes the universal service provider less aggressive in the price game when it competes with a firm that does not cover the whole set of markets (Valletti et al., 2002). In this paper, we fully characterize the resulting price equilibrium when universal service obligations are imposed. With a limited market coverage by the entrant or a small degree of degree of product differentiation, the equilibrium is a mixed strategy one.

JEL Codes: L13, L51

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1 Introduction

Along with the deregulation of most of the former public monopolies comes the question of universal service obligations (hereafter USO). How should we define USO? What are the costs of USO (Panzar, 2000, Rodriguez and Storer, 2000) and how should they be financed (Choné et al., 2002, Mirabel et al., 2009)? Which firms should be subject to USO (Hoernig, 2006)? What are the consequences of USO in the context of deregulation? These questions are at the heart of quite a vivid debate, both in the scientific and political cenacles. In the present paper, we aim at contributing to this debate by focusing on the last of the above mentioned questions. More precisely we offer an in-depth analysis of the nature of price competition in a deregulated industry subject to USO.

Valletti et al. (2002) underline the fact that whenever USO involve a constraint of uniform pricing, this constraint deeply alters the nature of price competition. More precisely, a uniform pricing constraint creates a strategic link between otherwise segmented markets and induces a less aggressive pricing pattern by the incumbent. Since prices are strategic complements, equilibrium prices tend to increase overall and this in turn is likely to affect the extent of entry by incoming firms. Anton et al. (2002) establish a comparable result under quantity competition.

The argument is best summarized as follows. Think of a reference industry consisting of a collection of segmented submarkets (typically, the industry for postal services, with submarkets corresponding to delivery routes in different geographical areas). Suppose then that the historical incumbent is challenged by an entrant on a limited number of submarkets. Assume further that USO constrain the incumbent's behavior: it must offer its services in all submarkets, at a uniform price. At the price competition stage, the incumbent's behavior is affected by the extent of the entrant's market coverage. If the entrant is a low scale competitor, the incumbent firm is better off setting a price close to the monopoly price in which case it enjoys near monopoly profits on the (relatively numerous) protected markets but possibly sells very little - or nothing- on the contested ones. If the entrant covers a larger set of submarkets, the incumbent is better off being more aggressive over the whole set of submarkets. In which case the profits lost on the protected markets are compensated by larger sales on the (relatively numerous) contested ones. Hence, by choosing the number of submarkets it challenges, the entrant controls the aggressiveness of the incumbent. Prices therefore decrease with the entrant's coverage (Valletti et al., lemma 1). For that reason, the entrant will strategically limit its entry scale.

As acknowledged by Valletti et al. (2002), this analysis is correct as long as the products sold by the incumbent and the entrant are sufficiently differentiated. In a more general analysis, we cannot exclude however that the incumbent's price best reply leaves him with no sales at all on the contested markets. It is then *as if* he withdrew on the protected ones. In these markets indeed it can charge the monopoly price and collect the corresponding monopoly profits. This strategy turns to be particularly attractive when competition is fierce on the contested markets (for example because products are close substitutes) and insulated markets are relatively numerous. Taking this strategy into account may destroy the above pure strategy Nash equilibrium. In other words, an almost immediate consequence of USO is actually to question the existence of a pure strategy Nash equilibrium (PSE). The possible lack of existence of PSE originates in the **possibility** for the incumbent to retreat on those markets which are not challenged by the entrant where it still benefits from a monopoly position. Taking this strategy into account makes the incumbent even softer in the price game than previously acknowledged.

In this paper, we assume that firms sell differentiated products and compete in prices. We show that for each possible market coverage possibly chosen by the entrant, there exists a degree of product differentiation below which the PSE does not exist. Unsurprisingly, the larger the extent of market coverage, the smaller the degree of differentiation below which a PSE fails to exist.[1] Then, we establish that when the existence of a PSE is problematic, a mixed strategy equilibrium (MSE) always exists. Moreover, we show that all prices in the support of the mixed strategies are strictly above those prices corresponding to the pure strategy candidate. Finally, we develop an example where we make the entrant's coverage endogenous and we show that at the optimal coverage, the resulting equilibrium in the price game is the mixed strategy one. In other words, USO indeed induce the entrant to control for the aggressiveness of the incumbent by limiting its coverage but this limitation is very likely to induce mixed equilibrium pricing. Since prices in a mixed equilibrium are strictly above the pure strategy ones, we may argue that the conclusions derived from models where the existence of a pure strategy equilibrium has been assumed systematically underestimate the negative consequences of USO on competition.

Finally, let us stress that the point we raise in this paper should not be viewed as a technical curiosity. To avoid the discontinuity in the incum-

[1] In the limit case where firms sell homogeneous products, a pure strategy equilibrium never exists (Hoernig, 2002)

bent's reaction function, one needs that products are so differentiated that the incumbent faces a positive demand at the monopoly price even if the competitor's good is available for free. Products like bulk mails, telecommunication services or energy are clearly not that differentiated. Moreover, in those markets, connecting consumers is costly[2] and, therefore, entrants do not necessarily cover the whole territory. Hence, the possibility of a MSE must be considered in all these industries.

Moreover, in these markets, consumers typically rely on a unique provider, at which they possibly buy several units, i.e. markets where benefiting from the service requires a form a affiliation and where there is no real benefits to be obtained from multiple affiliations. The Hotelling's location model is particularly well suited to model the discrete consumer's choice of an affiliation (See, for example, Laffont, Rey and Tirole (1998) for an application to telecommunication markets). In section 4, we develop an example based on the Hotelling set-up where there is no demand expansion after market opening. We show that, if the entrant's coverage is endogenous, the equilibrium in the price game is a MSE.

Likewise, in vertically differentiated product models, if, at some prices, the universal service provider is completely excluded from some local submarkets[3] then withdrawing on the protected market with the monopoly price is an attractive option for the incumbent. In such a case, the only valid equilibrium is the MSE.

2 Model and Benchmark Results

2.1 The Model

There is a continuum of identical local markets indexed by j, $j \in [0, N]$. USO consist of two constraints: a universal coverage constraint (UC) and an uniform price constraint (UP). If USO are imposed on the incumbent, firm I, this firm must serve all the markets at a uniform price p_i. The entrant, firm E, is not constrained by USO and serves a subset of the N markets at price p_e. Let n_e denote the index of the last market firm E has decided to compete in. The complete set of markets can thus be decomposed into two subsets: the set $[0, n_e]$ of contested markets and the complement $[n_e, N]$ of insulated ones.

[2] High connection costs justify the imposition of the universal coverage constraint.

[3] Armstrong (2008) develops a stylized postal model where the incumbent may loose all its clients in one region after market opening.

We assume that firm I and E sell differentiated products; they compete simultaneously in prices, taking $n_e > 0$ as given. Production costs are normalized to zero. Denote by $x_k^D(p_i, p_e)$ the (duopoly) demand addressed to firm $k = i, e$ in each of the contested markets. We also define a monopoly demand for each of the two firms, which we denote by $x_k^M(p_k)$. This demand is obviously relevant for the incumbent in the insulated markets but may also be relevant in contested markets whenever price differentials are such that the non-negativity constraint on $x_k^D(.)$ binds for one firm. These demands are assumed to be well behaved. In particular, there exists a unique well-defined monopoly solution and, in case of duopoly, goods are demand substitutes.

All costs are normalized to zero. The profit of firm $k = i, e$ on a market j is $\pi_k^D(p_i, p_e) = p_k x_k^D(p_i, p_e)$ or $\pi_k^M = p_k x_k^M(p_k)$ depending on whether firm k faces the duopoly or the monopoly demand on market j.[4]

2.2 Price competition without USO

Suppose first that there is no universal service constraint applying. Firm I can therefore price discriminate between insulated and contested markets. On the former, firm I applies the monopoly price $p_i^m \equiv \text{argmax}_{p_i} \pi_i^M(p_i)$. On the latter, each firm sets the profit maximizing prices. Optimal behavior is summarized in the following best reply functions:

$$\phi_i^1(p_e) \equiv \underset{p_i}{\text{argmax}}\, \pi_i^D(p_i, p_e) \tag{1}$$

$$\phi_e^1(p_i) \equiv \underset{p_e}{\text{argmax}}\, \pi_e^D(p_i, p_e) \tag{2}$$

Under suitable regularity assumptions on demand functions, these best reply functions are continuous and monotone. Let us further assume that there exists a unique Nash equilibrium (p_i^{1*}, p_e^{1*}) given by the solution to: $\{p_i = \phi_i^1(p_e),\ p_e = \phi_e^1(p_i)\}$. Without USO, the monopoly price p_i^m prevails on the insulated markets and equilibrium prices (p_i^{1*}, p_e^{1*}) applies on the contested ones. Customers in contested markets therefore face lower prices and a greater choice of products.

[4]Imposing USO is relevant when there are markets that otherwise would not be served. Usually this amounts to assume that serving a market j involves a fixed cost $g(j)$ and that there are unprofitable markets: for some $j \in [0, N]$, $\nexists p$ such that $\pi^M(p) - g(j) \geq 0$. As we concentrate mainly on the analysis of the price game, we will abstract from these fixed costs in the formal analysis.

3 Price competition with USO

Without USO, the incumbent's profit is additively separable between the n_e contested markets and the remaining $(N - n_e)$ insulated ones. This is no longer true under USO. The uniform pricing constraint creates a strategic link between the two types of markets because increasing market shares in the contested segment by decreasing the price involves an opportunity cost corresponding to those profits which are lost through this price decrease on the insulated markets. The characterization of a Nash equilibrium in prices is more involved because of this trade-off. We therefore develop the analysis in two steps. First, we (informally) discuss the structure of the incumbent's profit function under USO and explain why this structure leads to the possible non-existence of a pure strategy equilibrium. Second, we formally derive the shape of best reply correspondences and equilibria.

• Step 1:

Notice that, whatever p_e, the incumbent is always able to secure the monopoly profits on the insulated markets. In other words, firm I's $MinMax$ payoff equals to $(N - n_e)\pi_i^M(p_i^m)$. Notice also that p_i^m defines the upper bound for the price that the incumbent could possibly play in equilibrium.

Assume first that prices are such that each firm faces a positive demand in the contested markets while the incumbent is a monopolist in the insulated ones. The profit functions are defined by:

$$\Pi_i(p_i, p_e) = (N - n_e)\pi_i^M(p_i) + n_e\pi_i^D(p_i, p_e) \quad (3)$$

$$\Pi_e(p_i, p_e) = n_e\pi_e^D(p_i, p_e) \quad (4)$$

In price constellations for which demands are strictly positive, the best reply functions compute as $\phi_i(p_e, n_e) \equiv \text{argmax}_{p_i}\, \Pi_i(p_i, p_e)$ and $\phi_e(p_i, n_e) \equiv \text{argmax}_{p_e}\, \Pi_e(p_i, p_e) = \phi_e^1(p_i)$. Both π_i^D and π_i^M are concave in p_i so that they are each characterized by a unique local maximum: $\phi_i^1(p_e)$ and p_i^m respectively. Moreover, we have that $\phi_i^1(p_e) \leq p_i^m$. The best reply defined over their weighted sum $\phi_i(.)$ therefore exists and is unique. Moreover, this best reply involves price "bracketing": $\forall n_e \in [0, N]$, $\phi_i^1(.) \leq \phi_i(.) \leq p_i^m$.

The unique Nash equilibrium (p_i^*, p_e^*) in the price game obtains as the solution to $\{p_i = \phi_i(p_e, n_e), p_e = \phi_e^1(p_e)\}$. Notice further that the entrant's behavior is unaffected by its market coverage while the incumbent behaves more aggressively when n_e increases: $d\phi_i/dn_e < 0$. Since prices are strategic complements, it follows that equilibrium prices decrease as the entrant's market coverage increases.

The analysis just performed partly replicates the analysis proposed by Valletti et al., (2002). This analysis however is partly misleading. It fails

indeed to explicitly consider the case where the incumbent, when naming its monopoly price p_i^m actually retreats on the insulated markets because, given p_i its demand is exactly zero on the contested ones (i.e. the non-negativity constraint on $x_i^D(.)$ is strictly binding). When this is the case, the incumbent enjoys its $MinMax$ payoff. As a matter of fact, this $MinMax$ strategy might well be the incumbent's best reply against some relevant level of prices p_e.

More precisely, let us denote by $\hat{p}_i(p_e)$ the solution to equation $x_i^D(\hat{p}_i, p_e) = 0$, i.e. $\hat{p}_i(p_e)$ defines the critical price above which the incumbent faces no demand on the contested markets. Then, given p_e, the payoffs of the incumbent is formally defined by

$$\Pi_i(p_i, p_e) = \begin{cases} (N - n_e)\pi_i^M(p_i) + n_e\pi_i^D(p_i, p_e) & \text{if } p_i \leq \hat{p}_i(p_e) \\ (N - n_e)\pi_i^M(p_i) & \text{if } p_i \geq \hat{p}_i(p_e) \end{cases} \quad (5)$$

In this case, the incumbent's payoff is not globally concave anymore. Figure 1 illustrates this point. There are two local maximizers $\phi_i(.)$ and p_i^m. The extent to which the first maximizer dominates the second obviously depends on the extent of market coverage, i.e. on n_e. More fundamentally, the lack of concavity is likely to destroy the existence of a pure strategy equilibrium.

Notice that *a sufficient condition ensuring that this lack of concavity is not problematic consists in assuming that* $x_i^D(p_i^m, 0) > 0$. In this case indeed, the non-negativity constraint cannot be binding in the relevant domain of prices: since firm I will never quote a price above the monopoly price while firm E will not sell at loss. Valletti et al. (2002) implicitly assume that this condition is satisfied, which indeed can be interpreted on as putting a lower bound on the degree of product differentiation. In the analysis to follow we do not make such an assumption.

• Step 2: Once the lack of concavity in the incumbent's payoff is acknowledged, it follows that the incumbent's best reply could be either the monopoly price p_i^m or the local best reply $\phi_i(.)$ computed along the first branch of the profit function.

To characterize firm I's best reply, we must compare the payoffs along the two profiles:

$$n_e\pi_i^D(\phi_i, p_e) + (N - n_e)\pi_i^M(\phi_i) = (N - n_e)\pi_i^M(p_i^m) \quad (6)$$

Which can be rewritten as follows:

$$\frac{n_e}{N - n_e}\pi_i^D(\phi_i, p_e) + \pi_i^M(\phi_i) = \pi_i^M(p_i^m)$$

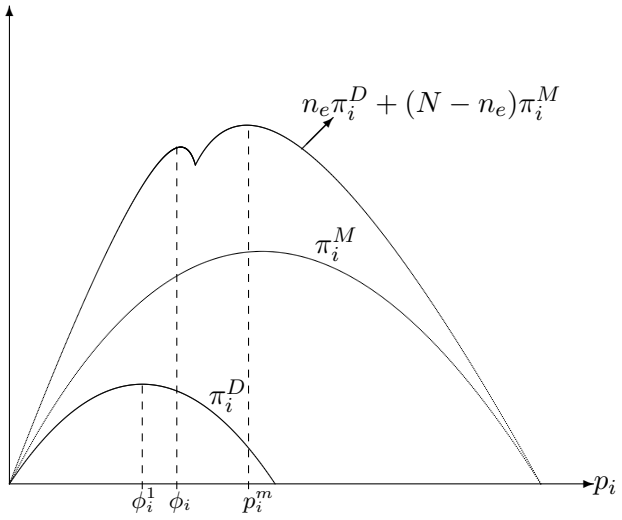

Figure 1: Non-concavity in the payoff of firm I

Because of strategic complementarity, the left-hand side of the equation is continuous and strictly increasing in p_e in the relevant domain whereas the right-hand side is constant. Moreover $\pi_i^M(\phi_i) \leq \pi_i^M(p_i^m)$. Accordingly, there exists at most one solution to the above equation. Let us denote this solution by $\tilde{p_e}$, i.e. $\tilde{p_e}$ defines the critical level of price for the entrant such that the incumbent is exactly indifferent between retreating on the insulated markets or being aggressive throughout. Obviously, for $p_e > \tilde{p}_e$, the incumbent is strictly better off being aggressive and challenges the entrant on all the n_e contested markets while for $p_e < \tilde{p}_e$, the incumbent is strictly better off securing its $MinMax$ payoff and withdrawing on the insulated markets.

The incumbent's best reply correspondence therefore writes as follows:

$$BR_i(p_e) \quad = \quad \begin{cases} p_i^m & \text{if } p_e \leq \tilde{p_e} \\ \phi_i(p_e, n_e) & \text{if } p_e \geq \tilde{p_e} \end{cases} \tag{7}$$

Since $p_i^m \geq \phi_i(\tilde{p_e}, n_e)$, the best reply correspondence exhibits a downward discontinuity at $\tilde{p_e}$.

As for the entrant's behavior, two strategy profiles are a priori possible. It can either compete with the incumbent on all contested markets or it can choose to quote a limit price, i.e. the highest possible price guaranteeing a monopoly position on the contested markets. The first strategy corresponds to $p_e = \phi_e^1(p_i)$, the second one to a limit price p_e^L defined as the solution

of $x_i^D(p_i, p_e) = 0$. The second strategy applies whenever the non negativity constraint is binding for firm I at prices $\phi_e^1(p_i)$. The entrant's best reply function is therefore kinked and defined as:

$$BR_e(p_i) = Max[\phi_e^1(p_i), p_e^L(p_i)] \tag{8}$$

3.1 Price equilibrium

There are a priori two pure strategy equilibrium candidates. First, there is the interior equilibrium candidate (p_i^*, p_e^*) where the incumbent challenges the entrant on all markets. Second, there is a "monopoly" equilibrium candidate, where the incumbent withdraws on the insulated markets and the entrant uses limit pricing to retain a monopoly position on the challenged markets, with prices p_i^m and $p_e^L(p_i^m)$ respectively. It is immediate to establish that the second candidate can be ruled out whenever the equilibrium monopoly price is interior, i.e. whenever the monopoly payoff function is differentiable at p_i^m.[5] In this case indeed, at p_i^m, the derivative of the payoffs on the insulated markets is zero whereas it is strictly negative on the contested ones. As a consequence, firm i's best reply must be $\phi_i(p_e^L, n_e)$. We are then left with a unique interior pure strategy equilibrium candidate. Because firm's I best reply is discontinuous, this equilibrium may not be a valid candidate either, as shown though the following Lemma.

Lemma 1 *Whenever $p_e^* < \tilde{p_e}$ there exists no pure strategy equilibrium*

This result is obvious when referring to Figure 2: if the downward jump occurs for a price $\tilde{p_e} > p_e^*$, then the pure strategy best reply of I against p_e^* is p_i^m. Accordingly, the pure strategy equilibrium does not exist.

The non-existence of a pure strategy equilibrium nicely summarizes the implication of USO on price competition. Suppose the entrant is very aggressive and names quite a low price. The incumbent is tempted to exploit its monopoly power over the insulated markets by naming the monopoly price. If this is the case, then the entrant should raise its price because competition is less fierce on the contested markets. But this, in turn, makes aggressiveness a profitable deviation for the incumbent. Because the incumbent is not allowed to price discriminate, these two opposite forces cannot equilibrate and price competition is fundamentally unstable.

[5] Notice that we provide in the next section an example where such a pure strategy equilibrium may exist.

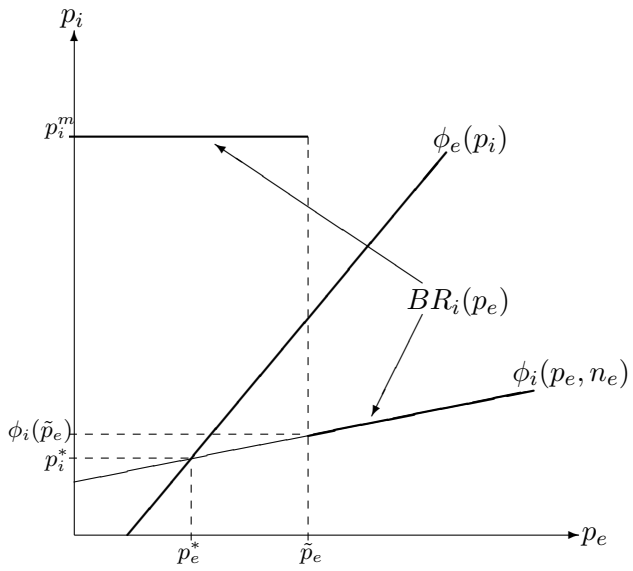

Figure 2: Non existence of a PSE

Lemma 2 *When a pure strategy does not exist, there always exists a mixed strategy equilibrium. In this equilibrium, average prices are strictly larger than the pure strategy equilibrium candidate* (p_i^*, p_e^*).

Proof: We prove this Lemma in two steps. First, as for the existence of a mixed strategy equilibrium, notice that because of product differentiation, firms' payoffs are continuous so that applying Glicksberg (1952)'s theorem, there always exists an equilibrium. In order to prove the second part of the Lemma let us denote by $\{F_k, [p_k^-, p_k^+)]\}$ the equilibrium strategy of firm k. Suppose, contrary to the Lemma that given p_e^-, the lowest price named by firm i in equilibrium satisfies $p_i^- < \phi_i(p_e^-)$. Then firm i's payoff must be strictly increasing at (p_i^-, p_e^-). But since p_e^- is the lowest price named by firm e in equilibrium, the same conclusion must hold for any other possible realization of the mixed strategy (p_i^-, p_e). This obviously implies that p_i^- is not part of firm i's equilibrium strategy. Since the exact same argument can be made regarding firm E's lowest price against p_i^-, we can conclude that in equilibrium $p_k^- \geq \phi_k(p_{-k}^-)$ for $k = i, e$. Notice then that this condition can be satisfied simultaneously for firm I and E if and only if $p_k^- \geq p_k^*$, which proves the Lemma. ■

From the above Lemma, it follows that either the equilibrium is the pure strategy one or if the pure strategy equilibrium does not exist, a non

degenerate mixed strategy equilibrium exists in which the pure strategies belonging to the support of equilibrium prices are larger than p_k^*. As a consequence, average prices realized in equilibrium are strictly above p_k^*. We have therefore established the following proposition.

Proposition 1 *The pure strategy equilibrium candidate (p_i^*, p_e^*) defines a lower bound to the level of prices named with positive probability in equilibrium.*

Notice that at this step, we do not have characterized a mixed strategy equilibrium of the game. This is a notoriously difficult task, which lies outside the scope of the present article. Nevertheless, we provide such a characterization for a simple example in the next section.[6]

3.2 Existence of a pure strategy Nash equilibrium

Equilibrium in the price game is either the pure strategy equilibrium or the mixed-strategy equilibrium with associated higher prices. It is therefore important to discuss which equilibrium applies. The answer to this question depends on the entrant's market coverage and the degree of product differentiation. When products are sufficiently differentiated, the PSE exists (Valletti et al., 2002 and Hoernig, 2006) while, for homogenous products, the unique equilibrium is the MSE whenever the incumbent has a strictly positive $MinMax$ payoff (Hoernig, 2002). Our aim is to identify the equilibrium for all possible degrees of product differentiation and coverage.

Let us assume that we can measure the degree of product differentiation by a parameter $\delta \in [\underline{\delta}, \overline{\delta}]$, with the lower bound corresponding to homogenous products and the higher bound to independent demands. We already established that a PSE always exists whenever $x_i^D(p_i^m, 0) > 0$ which implicitly defines a lower bound on δ above which existence is non-problematic. The following proposition characterizes the type of equilibrium prevailing for each possible value of δ and n_e.

Proposition 2 *(i) For each $n_e \in (0, N)$, there exists a degree of product differentiation $\tilde{\delta} < \overline{\delta}$ such that for $\delta \leq \tilde{\delta}$, the PSE fails to exist. (ii) $\tilde{\delta}$ is decreasing in n_e.*

Proof: The PSE fails to exist whenever $p_e^* \leq \tilde{p}_e$. Consider any given $n_e \in (0, N)$. When the degree of product differentiation δ varies, the equilibrium and the cut-off prices vary in opposite direction: $\frac{\partial p_e^*}{\partial \delta} > 0$ and

[6] Hoernig (2002) provides a characterization for the case of price competition with homogeneous goods.

$\frac{\partial \tilde{p}_e}{\partial \delta} < 0$. Moreover, when products are almost homogeneous, we have $p_e^* < \tilde{p}_e$: $\lim_{\delta \to \underline{\delta}} p_e^* = 0$ and $\lim_{\delta \to \underline{\delta}} \tilde{p}_e > 0$. Combined with the fact that whenever $x_i^D(p_i^m, 0) > 0$ the corresponding equilibrium is the PSE, we have proven part (i).

We have thus identified a locus $\tilde{\delta}(n_e)$ characterized by $p_e^*(\tilde{\delta}(n_e), n_e) = \tilde{p}_e(\tilde{\delta}(n_e), n_e)$. For a given δ, when $n_e > \tilde{\delta}^{-1}(n_e)$, the corresponding equilibrium is the PSE. As a matter of fact, both p_e^* and $\tilde{p}_e$ are decreasing in n_e and $\lim_{n_e \to N} p_e^* = p_e^{*1} > \lim_{n_e \to N} \tilde{p}_e = p_e^L(p_i^m))$. Therefore, for a given δ, the MSE applies for the lowest coverage and the PSE for the highest one. This proves that the locus $\tilde{\delta}(n_e)$ is decreasing in n_e. ■

Proposition 2 identifies for all possible degrees of product differentiation and for all possible coverage the corresponding equilibrium in the price game. Figure 3 illustrates the proposition. The PSE does not exist when the incumbent's $MinMax$ payoff is high (low coverage by the entrant) and when competition is fierce (little product differentiation). And, when the coverage increases (and thus, the incumbent's $MinMax$ payoff decreases), it is possible to sustain the PSE for more homogenous products.

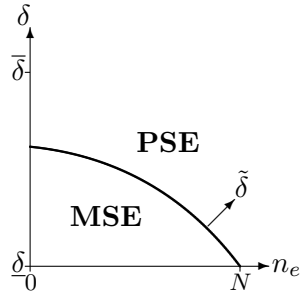

Figure 3: Equilibrium type in the price game

So far, we have considered that the entrant' coverage was exogenous. Valletti et al. (2002) show that when the entrant decides on its market coverage, a uniform price constraint induces a lower n_e. The reason is that a higher coverage intensifies competition and therefore reduces the equilibrium prices. It is immediate to show that the profit *on each covered market* j decreases with the number of covered markets: $\frac{\partial \pi_e(p_e^*, p_i^*)}{\partial n_e} < 0$. Hence, the marginal benefit of increasing market coverage is smaller than $\pi_e(p_e^*, p_i^*)$. Equilibrium coverage will be such that, on the last covered market, the marginal benefit equals the fixed cost of serving the market. Therefore, the entrant realizes a strictly positive profit on the last covered market, a result that would not be true absent the uniform pricing constraint.

Does a similar result continue to hold true when the MSE is considered? Since we do not characterize the mixed equilibrium in the general set-up, we are not able to answer this question formally. Decreasing market coverage leads to higher prices in equilibrium since the support of the mixed strategy equilibrium shifts up. This is likely to drive equilibrium profits up but we cannot prove this because we have no idea about the impact of decreasing market coverage on the equilibrium probability distributions.

In order to address the optimal coverage issue, we now develop an example where closed forms solutions can be computed. In this example, we show that the entrant's optimal coverage induces a pricing subgame in which only mixed strategy equilibria exist.

4 Application to Hotelling Competition

4.1 The model

At each market $j \in [0, N]$, consumers' type x are uniformly distributed in the $[0, 1]$ interval according to their idiosyncratic taste. The indirect utility derived from a consumer with type x, buying a product k is given by

$$U(x) = S - td(x, k) - p_j$$

where $d(x, k)$ is a measure of the distance between the product's characteristic and type x's ideal product. If the consumer does not buy any product, $U(x) = 0$.

The incumbent sells a product with type $x = 0$ at a constant marginal cost 0. Firm I is subject to USO and covers all the N markets at a uniform price p_i. The entrant possibly sells at price p_e a product with type $x = 1$ produced at a constant marginal cost 0. The entrant first decides on its market coverage $n_e \in [0, N]$. Then firms compete in prices.

We make the assumption that a monopolist will serve all the consumers. In each market, the monopoly payoff is given by $\pi_i^M = p_i \frac{S-p_i}{t}$ and this expression is maximized for $p_i = \frac{S}{2}$. We shall assume that $S > 2t$. As a result, the monopoly price on each market is a corner solution: $p_i^m = S - t$. The monopoly price leaves the consumer located at a distance 1 from the monopolist indifferent between buying and not buying. This assumption is perfectly in line with the literature on Hotelling competition, which most often assumes full coverage by assuming that S is *large enough*.

Notice that this assumption of monopoly market coverage amounts to assume that no market expansion effect is expected as a result of competition. All of the market shares gained by the entrant are taken from the

incumbent (the displacement ratio is equal to 1). As shown hereafter, this particular feature of the model reinforces that strategic value attached by the entrant to a voluntary limitation of the market coverage.

4.2 Price competition without USO

Let us consider first that the incumbent is allowed to price discriminate between contested and insulated markets. On the n_e contested markets, standard Hotelling competition takes places. Given consumers' preferences, the indifferent consumer is defined by $\tilde{x} = \frac{t-p_i+p_e}{2t}$ and firms I and E face a demand of $\hat{x}$ and $1-\hat{x}$ respectively. Each firm chooses the profit maximizing price:

$$\phi_i^1(p_e) \equiv \underset{p_i}{\operatorname{argmax}}\, p_i\tilde{x} = \frac{p_e+t}{2} \tag{9}$$

$$\phi_e^1(p_i) \equiv \underset{p_e}{\operatorname{argmax}}\, p_e(1-\tilde{x}) = \frac{p_i+t}{2} \tag{10}$$

The Nash equilibrium in the contested market is defined as the solution to (9) and (10): $p_i^{1*} = t = p_e^{1*}$. In the remaining $N-n_e$ markets, firm I remains as a monopolist and charge the monopoly price p_i^m.[7]

4.3 Price competition with USO

Under USO, the incumbent faces a positive demand on the contested markets whenever $p_i \leq p_e + t$. The profit of firm I defines as follow:

$$\Pi_i(p_i,p_e) = \begin{cases} (N-n_e)p_i & \text{if } p_i \geq p_e+t \\ (N-n_e)p_i + n_e p_i\tilde{x} & \text{if } p_i \leq p_e+t \end{cases} \tag{11}$$

The best reply correspondence is the following:

$$BR_i(p_e) = \begin{cases} p_i^m = S-t & \text{if } p_e \leq \tilde{p_e} \\ \phi_i(p_e,n_e) = t(\frac{N}{n_e}-1) + \frac{p_e+t}{2} & \text{if } p_e \geq \tilde{p_e} \end{cases} \tag{12}$$

Where $\tilde{p_e}$ is found by solving equation (6): $\tilde{p}_e = 1 - \frac{2N}{n_e} + \frac{2\sqrt{2}\sqrt{(N-n_e)n_e}\sqrt{S-t}}{n_e}$. Notice that, for a coverage close to zero, the price defined by $\phi_i(.)$ tends to increase exponentially. In which case consumers will stop buying and the

[7] When the fixed costs per market are all zero, the entrant would therefore covers all the N markets. When there are fixed costs of serving a market j, the coverage will be such that the net profit realized by E on the last covered market is zero.

demand would no longer be equal to $\tilde{x}$. Hence, the corresponding best reply must be computed as the solution to $U(\tilde{x}) = 0$ i.e. there is a third branch in (12) that applies against the highest values of p_e. Notice finally that in the present set-up, we also have that $p_e^L(p_i) = p_i - t$.

The profit realized by the entrant when it covers n_e markets is given by:

$$\Pi_e(p_i, p_e) = \begin{cases} n_e p_e & \text{if } p_e \leq p_i - t \\ n_e p_e(1 - \tilde{x}) & \text{if } p_e \geq p_i - t \end{cases} \tag{13}$$

And the best reply correspondence is the following:

$$BR_e(p_i) = Max[\phi_e^1(p_i), p_e^L(p_i)] \tag{14}$$

Depending on the entrant's coverage, there are three possible equilibrium configurations: two pure strategies equilibrium, $(p_i^m, p_e^L(p_i^m))$[8] and (p_i^*, p_e^*) and the mixed strategy equilibrium.

The first candidate is defined by ($p_i = p_i^m = S - t$, $p_e = p_e^L(p_i^m) = S - 2t$) in which case the incumbent monopolizes the protected markets and the entrant monopolizes the challenged markets with a limit price. A necessary condition for this candidate to be a valid one is that $\phi_i(p_e = S - 2t, n_e) \geq S - t$. This condition is satisfied whenever $n_e < n_e^- = \frac{2tN}{S+t}$. A second necessary condition is that $\phi_e^1(S - t) \leq S - 2t$, i.e the entrant's best reply against $S - t$ is not defined by the interior solution. This condition is satisfied whenever $S \geq 4t$. When these two conditions are satisfied, $(S - t, S - 2t)$ defines a Nash equilibrium. Moreover, this equilibrium is unique.[9]

The second candidate equilibrium in pure strategies can be identified by solving the system $\{p_i = \phi_i(p_e, n_e), p_e = \phi_e^1(p_i)\}$. We obtain:

$$p_i^* = \frac{t}{3}(\frac{4N}{n_e} - 1) \tag{15}$$

$$p_e^* = \frac{t}{3}(\frac{2N}{n_e} + 1) \tag{16}$$

This equilibrium applies whenever $p_e^* \geq \tilde{p}_e$ (lemma 1). This inequality defines a critical number of local markets n_e^+ above which the competitive pure strategy equilibrium exists (see figure 3). Formally n_e^+ is defined as:

$$n_e^+ = \frac{((2(6\sqrt{3}\sqrt{N^2(S-t)(-3+3S-t(7+t))} + N(9+18S-t(18+t))))}{(9+72S+(-78+t)t))}$$

[8] This equilibrium potentially exists because the monopoly price p_i^m is a corner solution.

[9] It is indeed immediate to show that the two pure strategy equilibrium candidates are mutually exclusive.

For intermediate value of $n_e \in [n_e^-, n_e^+]$ no pure strategy equilibrium exists. However, a mixed strategy equilibrium exists because payoffs are continuous (see lemma 2). Since only one of the two firms is subject to USO, it is natural to consider a candidate mixed strategy equilibrium in which only this firm uses a non-degenerate mixed strategy. Notice indeed that the kind of non-concavity in payoffs which results from USO (see figure 1) is present only in the incumbent's payoff function. In such an equilibrium, the incumbent randomizes over two prices: $p_i^+ = p_i^m$ with probability α and $p_i^- = \phi_i(\tilde{p_e}, n_e)$ with probability $1 - \alpha$. The probability is chosen to ensure that playing the pure strategy $\tilde{p_e}$ is indeed a best reply for firm E.[10]

Depending on the entrant's coverage, one of the following mutually exclusive equilibrium applies: The monopoly equilibrium $(p_i^m, p_e^L(p_i^m))$ for $n_e \in [0, n_e^-]$, the mixed strategy equilibrium for $n_e \in [n_e^-, n_e^+]$ and the interior equilibrium (p_i^*, p_e^*) for $n_e \in [n_e^+, N]$.

4.4 Coverage by the entrant

From equilibrium payoffs in the pricing game, it is direct to show that prices and the profit realized by the entrant in each covered market are weakly decreasing in n_e (and strictly decreasing for $n_e \geq n_e^-$). Therefore the entrant has strategic reasons to limit its market coverage. The following proposition establishes that, when coverage is endogenous and when there are no fixed costs of serving markets, the optimal coverage is such that the resulting price equilibrium is the mixed strategy one. If serving markets is costly, it will decrease further the entrant's coverage. Hence, in our example where local markets are represented by an Hotelling line, the interior equilibrium is never the relevant one.

Proposition 3 *The optimal coverage by the entrant n_e^* lies in the interval $[n_e^-, n_e^+]$*

Proof: (1) For $n_e \in [0, n_e^-]$, the entrant's profit $n_e(S-2t)$ is strictly increasing in n_e. (2) For $n_e \in [n_e^+, N]$, the entrant's profit $n_e p_e^*(1 - \tilde{x})$ is strictly decreasing in n_e. (3) The entrant's payoff is continuous in n_e. In particular, we have $\lim_{n_e \to n_e^+} \alpha = 0$ (since at n_e^+, $p_e^* = \tilde{p}_e$) and $\lim_{n_e \to n_e^-} p_i^- = p_i^+ = p_i^m$. Therefore, the highest payoff will be reached for a n_e in $[n_e^-, n_e^+]$. ■

To illustrate, assume that $N = 1$, $S = 5$, $t = 1$. Assuming that when a pure strategy does not exist, the mixed strategy equilibrium sees firm

[10] To the best of our knowledge, the structure of this equilibrium has been analyzed first by Krishna (1989) and developed afterwards in various contexts, see for instance Boccard and Wauthy (2003) for an application in the context of a Hotelling model.

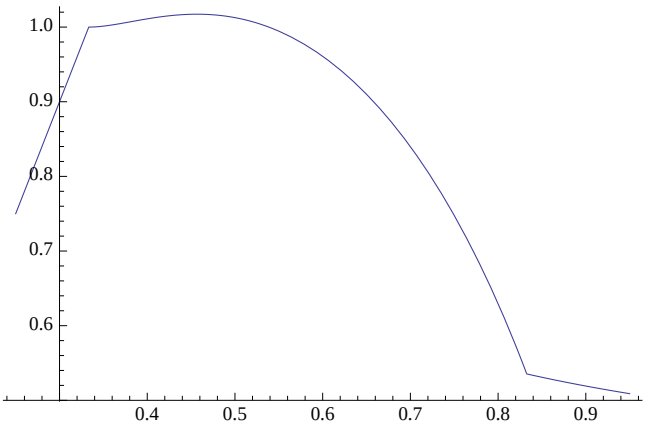

Figure 4: Entrant's equilibrium payoffs as a function of n_e

I mixing over two atoms and firm E playing a pure strategy, we obtain $n_e^- \simeq 0,33$, $n_e^+ \simeq 0,83$ and $n_e^* \simeq 0.45$. The entrant covers 45% of the markets. Figure 4 illustrates this numerical simulation and proposition 3.

Notice that in the present setup, the strategic effect of limited coverage is particularly neat: when limiting its market coverage, the entrant relaxes price competition which positively affects local profits and therefore possibly compensates for the smaller number of covered markets. But in addition, the equilibrium price differential between the incumbent and the entrant also increases. Since the displacement ration is equal to 1 (i.e. the market is fully covered), this implies that firm E's market share increases in all of the contested markets. This second effect clearly pushes incentives towards a more limited coverage.

5 Final Remarks

Imposing USO to one, or several firms operating in a formerly regulated industry raises many questions, some positive, other normative. In this paper we addressed a positive one: what is the impact of USO on the nature of price competition and thereby on the extent of market coverage by the entrant? This question is admittedly not a novel one and several important papers already tackle the issue, most notably Valletti et al. (2002). These papers emphasize the strategic links that results from the imposition of USO on the incumbent firms. USO weaken price competition because USO pe-

nalize the incumbent from fighting in the contested markets through the monopoly revenues lost on the protected ones. The entrant in turn may take benefit from this strategic link by controlling for the incumbent's aggressiveness through its own choice of market coverage. We argue however that the previous analysis did not push this nice intuition to its end by neglecting the fact that under low market coverage the willingness to retreat in the protected markets could actually lead to the non-existence of an equilibrium (in pure strategies).[11] In this paper, we show that this problem is almost a generic one: whatever the extent of market coverage, there exists products' characteristics for which the non-existence problem arises. We show that in a mixed strategy equilibrium, prices are higher on average. As a consequence, neglecting the existence of these mixed strategy equilibria amounts to underestimate the anti-competitive consequences of USO. Last, we show by mean of an example that the entrant's optimal coverage may indeed lead into the region where mixed strategy equilibria are part of the equilibrium outcome.

Our results are clearly partial ones. In particular, we have not been able to characterize completely the structure of mixed strategy equilibria. More research is called for on this topic and, according to our present analysis, this research would be particularly welcome to address the full implication of USO regulations on strategic choices made *before* price competition takes place, for instance the choice of products' attributes and/or quality that will be made available under USO.

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