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Centre de Recherche en Economie Publique et de la Population

CREPP WP No 2010/03

Endogenous Growth and Parental Funding of Education in an OLG Model with a Fixed Factor

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August 2010

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CREPP Working PaPer Series

Endogenous Growth and Parental Funding of Education in an OLG Model with a Fixed Factor*

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Abstract

This paper examines the stationary state income level and income growth in an overlapping generations (OLG) model in which production uses three inputs: physical capital, human capital and land. The accumulation of human capital relies on parental funding of education and the past aggregate human capital stock. Four cases exhibiting various possible specifications of returns to scale in output and human capital technologies are studied and compared.

Keywords: education, endogenous growth, increasing returns, human capital, land, overlapping generations.

JEL Classification numbers: E13, O11, O41.

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*I am grateful to David de la Croix, Klaus Desmet, Philippe Michel, Philippe Monfort and Henri Sneessens for helpful comments. Responsibility for all errors is my own.

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1 Introduction

In two-sector growth models, constant returns to scale technology applying to both sectors implies long-term growth when the reproducible factors, e.g. physical and human capital, grow at the same rate (Uzawa (1965), Lucas (1988), and Barro and Sala-I-Martin (1995) for a more general treatment of two-sector models). Human capital is generally defined as the skills acquired by workers through education or learning-by-doing (Becker 1964). For a given workforce, an increase in the aggregate human capital corresponds to an increase in the quality of this workforce, where quantity and quality are assumed to be perfect substitutes. In such models, human capital embodies technological progress in workers while physical capital remains a purely quantitative input. Therefore, nothing rules out theoretically that the long-term growth rates of physical and human capital be different. Moreover, technology departing from constant returns to scale offer a more general and richer analysis of growth.

More general specifications of the production functions for the final good and human capital have been considered in the literature. Mulligan and Sala-I-Martin (1993) study combinations of non-constant returns to scale technologies in a neoclassical growth model that are consistent with balanced growth in the long run. It is even possible to obtain increasing returns to scale in both production functions if we add a non-reproducible factor such as land or raw labor (or a combination of the two) in the output technology for example, and remain in a competitive framework. This is the modeling approach we adopt in this paper and apply it to a growth model with overlapping generations. Our objective is to consider a general specification for production functions in the sense that we use Cobb-Douglas functional forms in which the income shares of inputs may not sum up to one. In addition, we assume that technologies for the accumulation of human capital and the production of the final good are different. This assumption is generally justified by the empirical fact that human capital accumulation is relatively intensive in human capital. After determining the balanced growth condition along the stationary path, we compare the growth rates yielded by the combinations of non-constant returns to scale technologies that verify it. The main result of this paper shows that the economy exhibiting increasing returns in the final good and education sectors grow at a lower speed than the economy relying on constant returns to scale in both sectors.

The paper is organized as follows. Section 2 defines the model. Section 3 presents the effect of land when technologies are convex. Section 4 introduces non-convexities and determines the balanced growth condition. Section 5 compares the growth rates of different combinations of technologies. Finally, section 6 concludes.

2 The model

The model is an extension of the overlapping generations model of Allais (1947) and Diamond (1965).[1] The economy is closed and populated by overlapping generations, each living for three periods. The generation in period t is populated by N_t households and the total population grows at a constant rate n. When young, the individuals benefit from their parent's education spending and build their human capital. We therefore assume that there is no work neither explicit consumption during childhood. The consumption of children are thus included in their parents' consumption. When adult, the households work, consume and invest a part of their income in physical capital which is rented and used by the firms in the next period. They also devote part of their income to the accumulation of human capital which will benefit to their children. When old, they consume the return of their savings and die. In addition, each household owns a piece of land and a share of the firms. They thus receive land rents and profits. As there is no land market, they transmit their property rights over land to their children when they are old. As a result, only the working generation owns land. Each household is owner of the firms and receives interest on the capital rental. The firms buy inputs and produce the same single good in perfectly competitive markets. Each firm needs to locate its production activities on a piece of land. Therefore, land is a production factor and is priced at its marginal productivity that depends on the aggregate level of production. The single final good produced in this economy can either be consumed by the adult and the old generations or accumulated by the young households as capital for future production. The only consumption good is measured in units of final output.

2.1 Production technology

At each period the representative firm at the aggregate level produces a single good under a technology with constant or non-constant (social) returns to scale. There are three factors of production: physical capital, human capital and land. We assume that the production function of the representative firm is given by

$$Y_t = K_t^{\alpha} H_t^{\gamma} L^{\mu}, \quad 0 < \alpha, \gamma, \mu < 1 \tag{1}$$

where Y_t is the output, K_t is physical capital and H_t the stock of human capital used by the representative firm at time t. Physical capital is assumed to be fully depreciated after one period. L is the land endowment of this economy, which represents agricultural land or business estate where economic activities are located. It is assumed to be fixed over time and to enter the aggregate production function. The parameters α, γ and μ are the income shares or productivity elasticities of physical capital, human capital and land respectively. Each of these parameters are assumed to be strictly positive and strictly smaller than one. The problem of the firm is to maximize profits. Therefore, an

[1] The present model is a particular case of Artige (2010).

interior solution corresponding to a maximum of profits exists if the production function is concave, i.e., if the returns to scale with respect to the reproducible factors are non increasing:

$$\alpha + \gamma \leqslant 1 \tag{2}$$

2.2 Human capital

Human capital is assumed to be a productive and a reproducible asset, whose accumulation results from a technology that is different from the one applying to physical capital. Thus human capital is an imperfect substitute for consumption. Like physical capital, it is assumed that human capital is homogenous. The production function for the human capital accumulation is defined by

$$H_{t+1} = \Psi(N_t e_t)^{\theta} H_t^{\eta}, \quad \theta, \eta, \Psi > 0, \tag{3}$$

where θ and η are the elasticities of human capital accumulation with respect to education spending and to the past stock of human capital respectively, and Ψ is a scale technological parameter. The returns to scale of human capital accumulation are decreasing if $\theta + \eta < 1$, constant if $\theta + \eta = 1$, and increasing if $\theta + \eta > 1$. The stock of human capital at time $t+1$ is assumed to depend on contemporaneous aggregate education spending, $N_t e_t$, financed by the young adult generation, and on the human capital stock of the previous period, H_t. Education and inherited stock of human capital are thus imperfect substitutes. Moreover, each agent benefits from the aggregate human capital accumulated by the economy. It is therefore a positive externality that is beneficial to all agents. As in Uzawa (1965) and Lucas (1988), it is assumed that the production of human capital is relatively intensive in human capital but, unlike these authors, uses a positive quantity of physical capital (through education spending). An interior solution for an optimal choice of education spending is obtained if the private marginal return to investment in human capital is decreasing, i.e.:

$$\theta \leqslant 1 \tag{4}$$

2.3 Preferences

The representative consumer maximizes a logarithmic utility function of the type[2]

$$U(c_t, d_{t+1}, e_t) = \ln c_t + \beta \ln d_{t+1} + \lambda \ln e_t \tag{5}$$

[2]As in Andreoni (1989), de la Croix and Monfort (2000) or de la Croix and Michel (2002), this utility function includes an *ad hoc* altruism factor called "joy of giving". The parents' altruism for their children is not only a realistic assumption (see Abel and Warshawsky (1988)) but also allows to keep the modeling of the education financing very simple. In section 6, we use a different specification to model human capital accumulation.

subject to the following budget constraint,

$$\begin{aligned} c_t + s_t + e_t &= w_t \frac{H_t}{N_t} + \frac{\pi_t L}{N_t} \\ d_{t+1} &= R_{t+1} s_t \end{aligned}$$

Utility depends on consumption when young c_t, consumption when old d_{t+1}, and on the amount devoted to the offspring's education e_t. We therefore assume that the parents enjoy giving their children education resources as in Glomm and Ravikumar (1992). The parameter $\lambda > 0$ indicates the parents' degree of altruism. The parameter $\beta > 0$ is the psychological discount factor. The adults supply inelastically one unit of labor and earn $w_t H_t$, where w_t is the wage per unit of human capital and H_t is the aggregate level of human capital. They also receive $\pi_t L/N_t$ as land rent. Their income is allocated to consumption and saving, s_t, for future consumption. When old agents spend all their saving and accrued interest on consumption.

2.4 Profits

The maximization problem of the representative firm is defined by

$$\{K_t, H_t\} = \arg\max \{K_t^{\alpha} H_t^{\gamma} L^{\mu} - w_t H_t - R_t K_t - \pi_t L\}$$

where R_t is the interest factor and w_t the wage per unit of effective labor and π_t is the rent per unit of land. The representative firm maximizes its profits subject to the constraint of technology. Therefore, these profits depend on the technology. When returns to scale (social returns) are non-constant, profits are different from zero. In this case, we assume that (positive or negative) profits are redistributed to land owners. Therefore, π_t will represent the remuneration of the land factor and also the residual share in output (externality). Depending on the technology, the externality can be positive, negative or null.

2.5 Optimal behaviors

The representative consumer-producer chooses optimally c_t, e_t, d_{t+1} and H_t. As a representative firm, he hires the human capital input, H_t, according to (1). The human capital accumulates according to (2). As a representative consumer, he chooses c_t, d_{t+1}, e_t, and therefore, s_t, according to (4). Since profits reach a maximum by the concavity of the production function, the production factors are paid at their marginal productivities. Hence, the first order necessary conditions of the firm's program (1) are:

$$R_t = \alpha K_t^{\alpha-1} H_t^{\gamma} L^{\mu}, \tag{6}$$

$$w_t = \gamma K_t^\alpha H_t^{\gamma-1} L^\mu, \tag{7}$$

where R_t is the factor of interest and w_t is the wage per unit of effective labor. The young adult land owners receive the land rent equal to the marginal productivity of this factor and the residual income share:

$$\pi_t = (1-\alpha-\gamma) K_t^\alpha H_t^\gamma L^{\mu-1}. \tag{8}$$

The marginal productivity of land, $\frac{\partial Y}{\partial L}$, is positive. As for the residual income share, $\pi_t - \frac{\partial Y}{\partial L}$, it is negative when returns to scale are increasing ($\alpha+\gamma+\mu > 1$), it is positive when they are decreasing ($\alpha+\gamma+\mu < 1$) and it is null when they are constant ($\alpha+\gamma+\mu = 1$). The first order necessary conditions of the consumer's program (4) are:

$$s_t = \frac{\beta}{1+\beta+\lambda}\left(w_t \frac{H_t}{N_t} + \frac{\pi_t L}{N_t}\right), \tag{9}$$

$$e_t = \frac{\lambda}{1+\beta+\lambda}\left(w_t \frac{H_t}{N_t} + \frac{\pi_t L}{N_t}\right). \tag{10}$$

Saving and education spending are thus functions of the labor income and land rent. At the optimum, the relationship between education and saving is linear:

$$e_t = \frac{\lambda}{\beta} s_t. \tag{11}$$

2.6 Equilibrium

The equilibrium on the good market at period t is given by the national accounting identity:

$$Y_t = N_t c_t + I_t + N_t e_t, \tag{12}$$

where $N_t c_t$ is the aggregate consumption and $N_t e_t$ is the aggregate education expenditures at period t. The aggregate investment I_t is equal to the future physical capital stock K_{t+1} since the current capital stock K_t fully depreciates at the end of the current period. The equilibrium on the capital market derives from (12) and yields:

$$K_{t+1} = N_t s_t, \tag{13}$$

where $N_t s_t$ is the aggregate saving at period t. The dynamics will be analyzed in terms of three stationary variables: the physical-human capital ratio k_{t+1}, the growth factor of human capital $x_{t+1} = H_{t+1}/H_t$, and the growth factor of the economy $g_{t+1} = Y_{t+1}/Y_t$.

Equilibrium requires a stationary physical-human capital ratio that can be derived from the equilibrium interest factor (equation 6):

$$k_t \equiv \frac{K_t}{H_t^{\frac{\gamma}{1-\alpha}}}. \tag{14}$$

The marginal productivity of the production factors can be rewritten as:

$$\begin{aligned} R_t &= \alpha k_t^{\alpha-1} L^{\mu} \\ w_t &= \gamma k_t^{\alpha} H_t^{\frac{\gamma}{1-\alpha}-1} L^{\mu} \\ \pi_t &= (1-\alpha-\gamma) k_t^{\alpha} H_t^{\frac{\gamma}{1-\alpha}} L^{\mu-1} \end{aligned}$$

An equilibrium can now be characterized as follows: given initial conditions $\{K_0, H_0\}$ satisfying (13), an equilibrium is a vector of positive quantities $(K_t, H_t, c_t, d_t, s_t, e_t, \pi_t)_{t\geqslant 0}$ and prices $(R_t, w_t)_{t\geqslant 0}$ such that equations (1) to (14) hold. Equations (1) to (14) can be reduced to a system of three non-linear difference equations of the first order, describing the dynamics of the physical-human capital ratio k_t, the growth factor of human capital accumulation x_t and the growth factor of the economy g_t:

$$k_{t+1} = \frac{\beta}{(\Psi\lambda^{\theta})^{\frac{\gamma}{1-\alpha}}} \left(\frac{(1-\alpha)k_t^{\alpha}L^{\mu}}{1+\beta+\lambda}\right)^{1-\frac{\gamma\theta}{1-\alpha}} H_t^{\frac{\gamma}{1-\alpha}\left(1-\eta-\frac{\gamma\theta}{1-\alpha}\right)} \tag{15}$$

$$x_{t+1} = \frac{H_{t+1}}{H_t} = \Psi\left(\frac{\lambda(1-\alpha)k_t^{\alpha}L^{\mu}}{1+\beta+\lambda}\right)^{\theta} H_t^{\frac{\theta\gamma+\eta(1-\alpha)-1+\alpha}{1-\alpha}} \tag{16}$$

$$g_{t+1} = \frac{Y_{t+1}}{Y_t} = \frac{k_{t+1}^{\alpha}}{k_t^{\alpha}}(x_{t+1})^{\frac{\gamma}{1-\alpha}} \tag{17}$$

Equation (15) gives the dynamics of the physical-human capital ratio, equation (16) the growth factor of the human capital stock and equation (17) the growth factor of the economy. The growth rate of the income per capita is

$$\rho_{t+1} = \frac{y_{t+1}}{y_t} = \frac{1}{1+n}\frac{Y_{t+1}}{Y_t} = \frac{g_{t+1}}{1+n}, \tag{18}$$

where y_t is the income per capita at period t.

In the rest of the paper, we want to analyze growth paths of this economy using different combinations of technology applied to production and human capital accumulation.

3 Constant returns to scale (CRS) technology

In this section, it assumed that the technology is Cobb-Douglas for both the production function and the human capital accumulation. Thus, the production function (1) can be rewritten in the form of a Cobb-Douglas production function:

$$Y_t = K_t^{\alpha} H_t^{1-\alpha-\mu} L^{\mu}, \qquad \mu > 0 \tag{19}$$

where the factor income shares sum up to one and, hence, $\gamma = 1 - \alpha - \mu$. The production function of human capital (3) can be rewritten in the form of a Cobb-Douglas function:

$$H_{t+1} = \Psi (N_t e_t)^{\theta} H_t^{1-\theta}, \tag{20}$$

where $\eta = 1 - \theta$. Equilibrium requires a stationary physical-human capital ratio:

$$k_t \equiv \frac{K_t}{H_t^{\frac{1-\alpha-\mu}{1-\alpha}}}.$$

With Cobb-Douglas technologies, marginal productivities of production factors are as follows:

$$\begin{aligned} R_t &= \alpha k_t^{\alpha-1} L^{\mu} \\ w_t &= (1-\alpha-\mu) k_t^{\alpha} H_t^{\frac{-\mu}{1-\alpha}} L^{\mu} \\ \pi_t &= \mu k_t^{\alpha-1} H_t^{\frac{1-\alpha-\mu}{1-\alpha}} L^{\mu-1} \end{aligned}$$

The system of three non-linear difference equations (15)-(17) becomes:

$$k_{t+1} = \frac{\beta}{(\Psi\lambda^{\theta})^{\frac{1-\alpha-\mu}{1-\alpha}}} \left(\frac{(1-\alpha) k_t^{\alpha} L^{\mu}}{1+\beta+\lambda} \right)^{1-\frac{(1-\alpha-\mu)\theta}{1-\alpha}} H_t^{\frac{1-\alpha-\mu}{1-\alpha}\left(\frac{\mu\theta}{1-\alpha}\right)} \tag{21}$$

$$x_{t+1} = \frac{H_{t+1}}{H_t} = \Psi \left(\frac{\lambda(1-\alpha) k_t^{\alpha} L^{\mu}}{1+\beta+\lambda} \right)^{\frac{(1-\alpha-\mu)\theta}{1-\alpha}} H_t^{\frac{-\mu\theta}{1-\alpha}} \tag{22}$$

$$g_{t+1} = \frac{Y_{t+1}}{Y_t} = \frac{k_{t+1}^{\alpha}}{k_t^{\alpha}} (x_{t+1})^{\frac{1-\alpha-\mu}{1-\alpha}} \tag{23}$$

Equations (22) and (23) give the growth factors of human capital and aggregate income respectively.

3.1 Growth path in an economy with land and CRS technologies

From the system of equations (21)-(23), we can derive the following proposition:

Proposition 1 *In an OLG model with land and Cobb-Douglas technology (with factor shares summing up to one) in both production and human capital accumulation, the growth rate of income is zero in the long run.*

Proof:

For any $k_0 > 0$, $\frac{dx_{t+1}}{dH_t} < 0$ and $\lim_{H \to +\infty} x_{t+1} = 0$. Therefore, the growth rate of the economy in the long run is zero. Since the production technology is Cobb-Douglas and $\mu > 0$, the returns to scale of the reproducible factors in production are decreasing. Moreover, the returns to scale of the human capital accumulation are constant and do not compensate for the decreasing returns of production technology. The economy converges to a zero corner steady state

$$\bar{k}_1 = 0 \tag{24}$$

$$\bar{x}_1 = 0 \tag{25}$$

$$\bar{g}_1 = 0. \tag{26}$$

3.2 CRS technology: balanced growth condition

What is the condition for an economy with land exhibiting constant returns to scale technology in output and human capital to admit a balanced growth path, i.e., a state in which K, H, Y grow at a positive constant rate?

Proposition 2 *An OLG model with land and Cobb-Douglas technology (with factor shares summing up to one) in both production and human capital accumulation admits a balanced growth if and only if the income share of land is zero.*

Proof:

The economy admits a balanced growth path if and only if the growth rate (22) is equal to a constant. This happens when $H_t^{\frac{-\mu\theta}{1-\alpha}} = 1$. This condition is met if and only if $\mu = 0$.

The presence of non-reproducible factors offers the possibility to use non-convexities and externalities in a growth model. Note that, if the elasticity of increasing returns or externalities is lower than $\frac{(1-\alpha-\mu)\theta\mu}{1-\alpha}$, then the long run growth rate remains null. In developing countries where land still accounts for a large share of national product, externalities coming from knowledge in a broad sense, for example, may not be large enough for their economies to experience sustained growth.

3.3 Long-term growth without land

We assume that production and human capital accumulation exhibit constant returns to scale and the income elasticity of land, μ, is null. This is the standard OLG model with endogenous accumulation of human capital (see for instance d'Autume and Michel (1994)). The production function (1) can be rewritten in the form of a Cobb-Douglas production function where the output elasticities of production factors sum up to one:

$$Y_t = K_t^{\alpha} H_t^{1-\alpha}, \quad 0 < \alpha < 1,$$

where $\gamma = 1 - \alpha$. The production function of human capital remains as equation (20). Equilibrium requires a stationary physical-human capital ratio:

$$k_t \equiv \frac{K_t}{H_t}.$$

With Cobb-Douglas technologies, marginal productivities of production factors are as follows:

$$\begin{aligned} R_t &= \alpha k_t^{\alpha-1} \\ w_t &= (1-\alpha) k_t^{\alpha} \\ \pi_t &= 0 \end{aligned}$$

The system of three non-linear difference equations (15)-(17) becomes:

$$\begin{aligned} k_{t+1} &= \frac{\beta}{\Psi \lambda^{\theta}} \left(\frac{1-\alpha}{1+\beta+\lambda} \right)^{1-\theta} k_t^{\alpha(1-\theta)} \\ x_{t+1} &= \frac{H_{t+1}}{H_t} = \Psi \left(\frac{\lambda(1-\alpha)}{1+\beta+\lambda} \right)^{\theta} k_t^{\alpha\theta} \\ g_{t+1} &= \frac{Y_{t+1}}{Y_t} = \frac{k_{t+1}^{\alpha}}{k_t^{\alpha}} x_{t+1} \end{aligned}$$

Along the stationary path, the physical capital-human capital ratio is constant and physical capital and human capital (and hence income per capita) grow at the same positive constant factor. The values of the system at the stationary state are

$$\bar{k}_2 = \left[\frac{\beta}{\Psi \lambda^{\theta}} \left(\frac{1-\alpha}{1+\beta+\lambda} \right)^{1-\theta} \right]^{\frac{1}{1-\alpha(1-\theta)}} \quad (27)$$

$$\bar{g}_2 = \bar{x}_2 = \left[\Psi^{1-\alpha} \left(\frac{\beta^{\alpha} \lambda^{1-\alpha}(1-\alpha)}{1+\beta+\lambda} \right)^{\theta} \right]^{\frac{1}{1-\alpha(1-\theta)}}, \quad (28)$$

where $\bar{x}_2$ and $\bar{g}_2$ are positive constants.

4 Non-constant returns to scale (Non-CRS) technology

Let us now consider an economy with land exhibiting non-constant returns to scale to production and human capital accumulation. We use Cobb-Douglas functional forms but the output elasticities of inputs may not sum up to one. Our interest focuses on any combination of production and human capital technologies satisfying the balanced growth path condition.

4.1 Non-CRS technology: balanced growth condition

Equation (17) gives the growth factor of the economy and allows us to derive the balanced growth condition.

Proposition 3 *An OLG model with land and human capital, exhibiting non-constant returns to scale technologies, admits a balanced growth path for conditional values.*

Proof:

A balanced growth path exists if and only if the growth factor of the economy g_{t+1} of equation (17) is equal to a constant, which is the case if the growth factor of human capital x_{t+1} of equation (16) is also equal to a constant. This requires that

$$\frac{\theta\gamma + \eta(1-\alpha) - 1 + \alpha}{1-\alpha} = 0$$

i.e.,

$$\frac{\gamma}{1-\alpha} = \frac{1-\eta}{\theta}. \quad (29)$$

Equation (29) allows for three cases:

i) $\alpha + \gamma < 1$ and $\theta + \eta > 1$;

ii) $\alpha + \gamma = 1$ and $\theta + \eta = 1$;

iii) $\alpha + \gamma > 1$ and $\theta + \eta < 1$.

Case iii) is ruled out by assumption (2). Therefore, the balanced growth condition (29) allows for two possible growth regimes. Interestingly, this condition can be interpreted in terms of growth rates of the reproducible factors. In fact, along the balanced growth path,

$$\frac{Y_{t+1}}{Y_t} = \left(\frac{K_{t+1}}{K_t}\right)^{\alpha} \left(\frac{H_{t+1}}{H_t}\right)^{\gamma}.$$

Since the capital stock K is equal to aggregate saving, i.e. a fraction of the output Y, then along the balanced growth path, the income growth factor is equal to the growth factor of the physical capital. This implies,

$$g_{t+1} = (x_{t+1})^{\frac{\gamma}{1-\alpha}}. \tag{30}$$

If $\gamma < 1 - \alpha$, i.e. if returns to scale to reproducible factors in the production function are decreasing, the growth rate of human capital accumulation is higher than the growth rate of the economy and lower otherwise.[3] In the sector of the production of human capital, the growth factor of the human capital stock must be constant along the balanced growth path and yields

$$\left(\frac{H_{t+1}}{H_t}\right)^{1-\eta} = \left(\frac{e_{t+1}}{e_t}\right)^{\theta}.$$

Since education spending is a fraction of output, then, along the balanced growth path,

$$g_{t+1} = (x_{t+1})^{\frac{1-\eta}{\theta}}. \tag{31}$$

Equality (31) is not necessarily the same as equality (30). The condition for a balanced growth path (29) states that the growth factors of physical capital and human capital must be identical in the output and human capital production sectors.

[3] It could be possible to have increasing returns to scale (social returns) to reproducible factors in the production function ($\gamma + \alpha > 1$) in a setting with knowledge spillovers that firms could not internalize. Therefore, private returns to reproducible factors would be constant while social returns would be increasing. In our setting, firms internalize all returns.

4.2 Two balanced growth regimes

The balanced growth condition (29) allows to consider two growth regimes for an economy with land and human capital. The first corresponds to increasing returns to scale in both production and human capital accumulation. In the second regime, returns to scale are increasing in production and constant in human capital accumulation.

4.2.1 Regime 1: increasing returns to scale in production and human capital accumulation

In this growth regime, we consider an OLG model with land exhibiting increasing returns to scale in output and human capital technologies. The production technology of the firm is defined by:

$$Y_t = K_t^{\alpha} H_t^{\gamma} L^{\mu},$$

where the elasticities of the reproducible factors $\alpha + \gamma < 1$ and $\mu > 0$. However, the sum of the factor elasticities $\alpha + \gamma + \mu$ can be higher than 1, which yields increasing returns to scale in production. Human capital accumulates according to

$$H_{t+1} = \Psi e_t^{\theta} H_t^{\eta}, \quad 0 < \theta < 1.$$

where it is assumed that $\theta + \eta > 1$. The returns to scale of human capital accumulation are thus increasing. Equilibrium requires a stationary physical-human capital ratio:

$$k_t \equiv \frac{K_t}{H_t^{\frac{\gamma}{1-\alpha}}}.$$

Marginal productivities of production factors are as follows:

$$\begin{aligned} R_t &= \alpha k_t^{\alpha-1} L^{\mu} \\ w_t &= \gamma k_t^{\alpha} H_t^{\frac{\gamma}{1-\alpha}-1} L^{\mu} \\ \pi_t &= (1-\alpha-\gamma) k_t^{\alpha} H_t^{\frac{\gamma}{1-\alpha}} L^{\mu-1} \end{aligned}$$

The variables R_t, w_t, π_t are the equilibrium factor prices per unit of inputs K_t, H_t and L. The system of three non-linear difference equations (15)-(17) admits a balanced growth path if condition (29) is satisfied. Then, the system can rewritten as

$$k_{t+1} = \frac{\beta}{(\Psi\lambda^{\theta})^{\frac{\gamma}{1-\alpha}}} \left(\frac{(1-\alpha)L^{\mu}}{1+\beta+\lambda}\right)^{\eta} k_t^{\alpha\eta}$$

$$x_{t+1} = \frac{H_{t+1}}{H_t} = \Psi \left(\frac{\lambda(1-\alpha)L^{\mu}}{1+\beta+\lambda}\right)^{\theta} k_t^{\alpha\theta}$$

$$g_{t+1} = \frac{Y_{t+1}}{Y_t} = \frac{k_{t+1}^{\alpha}}{k_t^{\alpha}} (x_{t+1})^{\frac{\gamma}{1-\alpha}}$$

Along the balanced growth path, the stock of physical capital per effective unit of labor and the growth factor of the economy are constant:

$$\bar{k}_3 = \left(\frac{\beta}{(\Psi\lambda^{\theta})^{\frac{\gamma}{1-\alpha}}} \left(\frac{(1-\alpha)L^{\mu}}{1+\beta+\lambda}\right)^{\eta}\right)^{\frac{1}{1-\alpha\eta}} \quad (32)$$

$$\bar{x}_3 = \left[\Psi^{(1-\alpha)(1-\alpha\eta)-\gamma\alpha\theta} \left(\frac{\beta^{\alpha}\lambda^{1-\alpha}(1-\alpha)L^{\mu}}{1+\beta+\lambda}\right)^{\theta}\right]^{\frac{1}{1-\alpha\eta}} \quad (33)$$

$$\bar{g}_3 = (\bar{x}_3)^{\frac{\gamma}{1-\alpha}}, \quad (34)$$

where $\alpha + \gamma < 1$ and $\eta > 1 - \theta$ and $\frac{\gamma}{1-\alpha} = \frac{1-\eta}{\theta}$. Equation (32) shows that the stationary physical capital stock per effective unit of labor $\bar{k}_3$ increases with the elasticity of the human capital externality η provided that the expression within the main brackets of (32) is higher than 1. The presence of a non-reproducible factor, L, in the production function allows for increasing returns to scale in human capital accumulation and, hence, for a larger effect of the internalized human capital externality on the stationary income level. However, due to the concavity of the production function, a higher $\bar{k}$ caused by a higher η result in a lower income growth rate along the stationary state.

If we fix the values of α and η, then we have θ as a function of $1/\gamma$.

4.2.2 Regime 2: Increasing returns to scale in production and constant returns to scale in human capital accumulation

In this third growth regime, we consider an OLG model with land, in which the sum of the elasticities of the reproducible factors in the production technology and in human capital accumulation sum up to one. The production technology of the firm is defined by:

$$Y_t = K_t^{\alpha} H_t^{1-\alpha} L^{\mu},$$

where the elasticities of the reproducible factors $\alpha+\gamma=1$ and $\mu>0$. However, the sum of the factor elasticities, $\alpha+\gamma+\mu>1$, yields increasing returns to scale in production. Human capital accumulation technology is

$$H_{t+1}=\Psi e_t^{\theta}H_t^{1-\theta}, \quad 0<\theta<1.$$

where $\eta=1-\theta$. The returns to scale of human capital accumulation are thus constant. Equilibrium requires a stationary physical-human capital ratio:

$$k_t\equiv\frac{K_t}{H_t^{\frac{\gamma}{1-\alpha}}}=\frac{K_t}{H_t}.$$

Marginal productivities of production factors are as follows:

$$\begin{aligned}R_t &= \alpha k_t^{\alpha-1}L^{\mu}\\ w_t &= (1-\alpha)k_t^{\alpha}L^{\mu}\\ \pi_t &= 0\end{aligned}$$

The marginal productivity of land is positive but offset by the residual income share. Therefore, the return to land is null. The system of two non-linear difference equations (15)-(17) becomes:

$$k_{t+1} = \frac{\beta}{(\Psi\lambda^{\theta})^{\frac{\gamma}{1-\alpha}}}\left(\frac{(1-\alpha)L^{\mu}}{1+\beta+\lambda}\right)^{1-\theta}k_t^{\alpha(1-\theta)} \tag{35}$$

$$x_{t+1} = \frac{H_{t+1}}{H_t}=\Psi\left(\frac{\lambda(1-\alpha)L^{\mu}}{1+\beta+\lambda}\right)^{\theta}k_t^{\alpha\theta} \tag{36}$$

$$g_{t+1} = \frac{Y_{t+1}}{Y_t}=\frac{k_{t+1}^{\alpha}}{k_t^{\alpha}}x_{t+1} \tag{37}$$

Equations (36) and(37) show that the system (35)-(37) admits a balanced growth path when k reaches the stationary state. Since the elasticities of the reproducible factors sum up to one and the returns to human capital accumulation are constant ($\theta=1-\eta$), per capita income grows linearly. Along the balanced growth path, the stock of physical capital and the growth rate of the economy are positive constants:

$$\begin{aligned}\bar{k}_4 &= \left[\frac{\beta}{\Psi\lambda^{\theta}}\left(\frac{(1-\alpha)L^{\mu}}{1+\beta+\lambda}\right)^{1-\theta}\right]^{\frac{1}{1-\alpha(1-\theta)}}\\ \bar{g}_4 &= \bar{x}_4=\left[\Psi^{1-\alpha}\left(\frac{\beta^{\alpha}\lambda^{1-\alpha}(1-\alpha)L^{\mu}}{1+\beta+\lambda}\right)^{\theta}\right]^{\frac{1}{1-\alpha(1-\theta)}}\end{aligned}$$

As before, the stationary physical capital stock per effective unit of labor $\bar{g}_4$ increases with the elasticity of the human capital externality η, but as $(\theta = 1 - \eta)$, it decreases with the elasticity of education spending. As for the income growth rate, $\bar{g}_4$, it increases with the elasticity of education spending or decreases with the elasticity of the human capital externality. Compared with case 2, the share of land in output, μ, is no longer a problem for sustained growth. By benefitting from overall steady productivity, land affects positively the growth rate of the economy.

5 Comparison of growth regimes

The presence of a fixed factor allows for different combinations of technologies in production and human capital accumulation. We now want to compare the growth factors yielded by these different regimes.

Proposition 4 *In an OLG model with human capital exhibiting constant returns to scale, the growth factor of the model without land, g_2, is lower than that of the model with land , g_4.*

Proof:

It is straightforward to prove that $g_2 < g_4$. The expressions of g_2 and g_4 are identical except for the presence of the land factor L^μ in g_4. If $L^\mu > 1$, then $g_2 < g_4$.

Proposition 5 *In an OLG model with human capital and land exhibiting non-constant returns to scale, the growth factor of the model with increasing returns in both production and human capital accumulation, g_3, is lower than that of the model with increasing returns in production and constant returns in human capital accumulation, g_4.*

To prove that $g_3 < g_4$, we fix the values of α and θ and let the values of γ and η differ between both regimes. In the regime of g_3, the balanced growth condition (29) must be satisfied:

$$\frac{\gamma_3}{1-\alpha} = \frac{1-\eta_3}{\theta} = \phi. \tag{38}$$

Since $\gamma_3 < 1 - \alpha$, $\phi < 1$. We can rewrite g_3 as

$$g_3 = (\Omega)^{\frac{\phi}{1-\alpha(1-\theta\phi)}}, \tag{39}$$

where $\Omega = \Psi^{(1-\alpha)(1-\alpha\eta)-\gamma\alpha\theta} \left(\frac{\beta^\alpha \lambda^{1-\alpha}(1-\alpha)L^\mu}{1+\beta+\lambda}\right)^\theta$. Let us assume that $\Psi = 1$. Then,

$$\Omega = \left(\frac{\beta^{\alpha}\lambda^{1-\alpha}(1-\alpha)L^{\mu}}{1+\beta+\lambda} \right)^{\theta}$$

Let us now write the growth factor g_4 as a function of Ω:

$$g_4 = x_4 = (\Omega)^{\frac{1}{1-\alpha(1-\theta)}}. \tag{40}$$

Rearranging the power in the expression (39), we can show that

$$\frac{1}{\frac{1}{\phi}(1-\alpha)+\alpha\theta} < \frac{1}{(1-\alpha)+\alpha\theta} \tag{41}$$

and conclude that $g_3 < g_4$.

The accumulation of human capital depends on the aggregate education spending, the past aggregate stock of human capital and their elasticities in the accumulation rule. While the sum of the elasticities in the accumulation of human capital is higher in g_3 than in g_4, the elasticity of human capital in the production function is lower in g_3 than in g_4. As education spending is a fraction of income, a marginal increase in physical capital or human capital yields a lower marginal increase in education spending in g_3 than in g_4. In total, in the growth regime of g_3, it turns out that the increasing returns in the human capital accumulation does not sufficiently compensate for the decreasing returns to the reproducible factors in production to reach the level of g_4.

6 Conclusion

This paper studies balanced growth in an overlapping generations model with human capital and a fixed factor (land). The presence of a fixed factor allows for increasing returns to scale in production. We derived the balanced growth condition and identified the combination of technologies of both sectors yielding the highest rate of growth along the balanced growth path.

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