

# Interest parity condition

Lionel Artige

HEC – Université de Liège

5 December 2010

Interest parity condition: nominal values

- Let  $a_t$  and  $b_t$  be the prices of a dollar asset and a euro asset respectively at time  $t$ .
- $r_a$  and  $r_b$  are the annual interest rate on the dollar asset and the euro asset respectively.

- After one year the holder of the dollar asset receives:

$$a_{t+1} = (1 + r_a) a_t$$

- After one year the holder of the euro asset receives:

$$b_{t+1} = (1 + r_b) b_t$$

To compare the returns of the two assets, it is necessary to express their values in the same currency. To do so, we use the market exchange rate:

$$b_t = e_t \times a_t$$

where  $e_t$  is the number of euros for 1 dollar.  
This is the nominal exchange rate.

- After one year the holder of the euro asset receives in euro terms:

$$b_{t+1} = (1 + r_b) b_t$$

- or in dollar terms:

$$e_{t+1} \cdot a_{t+1} = (1 + r_b) e_t \cdot a_t$$

- After one year the holder of the dollar asset receives:

$$a_{t+1} = (1 + r_a) a_t$$

- After one year the holder of the euro asset receives in dollar terms:

$$e_{t+1} \cdot a_{t+1} = (1 + r_b) e_t \cdot a_t$$

- Interest parity condition requires that the rate of returns of both assets are equal once they have been expressed in the same currency:

$$1 + r_b = (1 + r_a) \cdot (e_{t+1} / e_t)$$

- which is equivalent to:

$$1 + r_b = (1 + r_a) \cdot \{[(e_{t+1} - e_t) / e_t] + 1\}$$

- And finally:

$$r_b - r_a = [(e_{t+1} - e_t) / e_t] + r_a \cdot [(e_{t+1} - e_t) / e_t]$$

- The last term on the right-hand side can be considered as negligible if the interest rate and the rate of change of the nominal exchange rate are less than 0.05.



- Let now  $a_t$  and  $b_t$  be the units of a dollar asset and a euro asset respectively at time  $t$  and  $r_a$  and  $r_b$  be the annual real interest rate on the dollar asset and the euro asset respectively.
- Let be  $p_{a,t}$  and  $p_{b,t}$  the prices of the dollar and the euro assets at time  $t$ .

Interest parity condition: real values

- What is the variation of the nominal value of the dollar asset between  $t$  and  $t+1$  ?

$$\frac{p_{a,t+1}a_{t+1}}{p_{a,t}a_t} = \frac{p_{a,t+1}}{p_{a,t}}(1 + r_a)$$

- What is the variation of the nominal value of the euro asset between  $t$  and  $t+1$  ?

$$\frac{p_{b,t+1}b_{t+1}}{p_{b,t}b_t} = \frac{p_{b,t+1}}{p_{b,t}}(1 + r_b)$$

Interest parity condition: real values

- What is the variation of the nominal value of the euro asset in dollar terms between  $t$  and  $t+1$  ?

$$\frac{P_{a,t+1}a_{t+1}}{P_{a,t}a_t} = \frac{P_{b,t+1}}{P_{b,t}}(1 + r_b)\frac{e_t}{e_{t+1}}$$

where  $e_t$  is the nominal exchange rate at time  $t$  as previously.

Interest parity condition: real values

- Interest parity condition requires that the returns of both assets are equal:

$$\frac{P_{a,t+1}}{P_{a,t}}(1 + r_a) = \frac{P_{b,t+1}}{P_{b,t}}(1 + r_b)\frac{e_t}{e_{t+1}}$$

which yields ...

Interest parity condition: real values

... the real interest parity condition:

$$\frac{1 + r_b}{1 + r_a} = \frac{\epsilon_{t+1}}{\epsilon_t}$$

where  $\epsilon_t$  is the real exchange rate at time  $t$ :

$$\epsilon_t = e_t \times \frac{p_{a,t}}{p_{b,t}}$$